

Solving Linear Systems of Equations

Numeric Methods: A system of equations is solved numerically using a **Given/Find** solve block. Mathcad requires seed values for each of the variables in the numeric method.

Seed values: $x := 1 \quad y := 1 \quad z := 1$

Given $5 \cdot x + 2 \cdot y + z = 36 \quad x + 7 \cdot y + 3 \cdot z = 63 \quad 2 \cdot x + 3 \cdot y + 8 \cdot z = 81$

$$\text{Find}(x, y, z) = \begin{pmatrix} 3.6 \\ 5.4 \\ 7.2 \end{pmatrix}$$

Other Given/Find solve blocks can be used:

$$\text{Given} \begin{pmatrix} 5 \cdot x + 2 \cdot y + z \\ x + 7 \cdot y + 3 \cdot z \\ 2 \cdot x + 3 \cdot y + 8 \cdot z \end{pmatrix} = \begin{pmatrix} 36 \\ 63 \\ 81 \end{pmatrix} \quad \text{Find}(x, y, z) = \begin{pmatrix} 3.6 \\ 5.4 \\ 7.2 \end{pmatrix}$$

$$\text{Given} \begin{pmatrix} 5 & 2 & 1 \\ 1 & 7 & 3 \\ 2 & 3 & 8 \end{pmatrix} \cdot \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 36 \\ 63 \\ 81 \end{pmatrix} \quad \text{Find}(x, y, z) = \begin{pmatrix} 3.6 \\ 5.4 \\ 7.2 \end{pmatrix}$$

Matrix methods: The equations can also be solved using matrix algebra as shown below. In matrix form, the equations are written as $M \cdot X = C$. The solution vector is found by matrix multiplication of C by the inverse of M :

$$M := \begin{pmatrix} 5 & 2 & 1 \\ 1 & 7 & 3 \\ 2 & 3 & 8 \end{pmatrix} \quad C := \begin{pmatrix} 36 \\ 63 \\ 81 \end{pmatrix} \quad X := M^{-1} \cdot C \quad X = \begin{pmatrix} 3.6 \\ 5.4 \\ 7.2 \end{pmatrix}$$

Confirm that a solution has been found:

$$M \cdot X = \begin{pmatrix} 36 \\ 63 \\ 81 \end{pmatrix}$$

Alternative matrix solution using the Isolve command:

$$X := \text{Isolve}(M, C) \quad X = \begin{pmatrix} 3.6 \\ 5.4 \\ 7.2 \end{pmatrix} \quad M \cdot X = \begin{pmatrix} 36 \\ 63 \\ 81 \end{pmatrix}$$

Live symbolic method:

To use the live symbolic method within this Mathcad document requires recursive definitions to clear previous values of x, y and z. This would not be necessary if x, y and z had not been previously defined.

$$x := x \quad y := y \quad z := z$$

$$\begin{pmatrix} 5 \cdot x + 2 \cdot y + z = 36 \\ x + 7 \cdot y + 3 \cdot z = 63 \\ 2 \cdot x + 3 \cdot y + 8 \cdot z = 81 \end{pmatrix} \text{ solve, } \begin{pmatrix} x \\ y \\ z \end{pmatrix} \rightarrow \left(\frac{18}{5} \quad \frac{27}{5} \quad \frac{36}{5} \right) = (3.6 \quad 5.4 \quad 7.2)$$