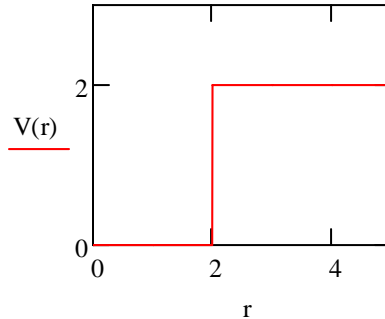


## Variation Method for a Particle in a Finite Spherical Potential

This problem deals with a particle of unit mass in a finite spherical potential well of radius  $2 a_0$  and well height  $2 E_p$ . The trial wave function is given below.

$$\Psi(r, \beta) := \left(\frac{2 \cdot \beta}{\pi}\right)^{\frac{3}{4}} \cdot \exp(-\beta \cdot r^2) \quad T = -\frac{1}{2 \cdot r} \cdot \frac{d^2}{dr^2}(r \cdot \Psi) \quad V(r) := \text{if}[(r \leq 2), 0, 2]$$



a. Demonstrate that the wave function is normalized.

$$\int_0^{\infty} \Psi(r, \beta)^2 \cdot 4 \cdot \pi \cdot r^2 \, dr \quad \left| \begin{array}{l} \text{assume, } \beta > 0 \\ \text{simplify} \end{array} \right. \rightarrow 1$$

b. Evaluate the variational integral.

$$E(\beta) := \int_0^{\infty} \Psi(r, \beta) \cdot \left[ -\frac{1}{2 \cdot r} \cdot \frac{d^2}{dr^2}(r \cdot \Psi(r, \beta)) \right] \cdot 4 \cdot \pi \cdot r^2 \, dr \dots \quad \left| \begin{array}{l} \text{assume, } \beta > 0 \\ \text{simplify} \end{array} \right. \quad \blacksquare$$

$$+ \int_2^{\infty} 2 \cdot \Psi(r, \beta)^2 \cdot 4 \cdot \pi \cdot r^2 \, dr$$

$$E(\beta) := \frac{1}{2} \cdot \frac{3 \cdot \pi^{\frac{1}{2}} \cdot \beta + 4 \cdot \pi^{\frac{1}{2}} + 16 \cdot \exp(-8 \cdot \beta) \cdot 2^{\frac{1}{2}} \cdot \beta^{\frac{1}{2}} - 4 \cdot \pi^{\frac{1}{2}} \cdot \text{erf}\left(2 \cdot 2^{\frac{1}{2}} \cdot \beta^{\frac{1}{2}}\right)}{\pi^{\frac{1}{2}}}$$

c. Minimize the energy with respect to the variational parameter  $\beta$ .

$$\beta := .5 \quad \beta := \text{Minimize}(E, \beta) \quad \beta = 0.381 \quad E(\beta) = 0.786$$

d. Calculate the average value of  $r$ .

$$\int_0^{\infty} r \cdot \Psi(r, \beta)^2 \cdot 4 \cdot \pi \cdot r^2 \, dr = 1.293$$

e. Calculate the kinetic and potential energy.

Potential energy:  $\int_2^{\infty} 2 \cdot \Psi(r, \beta)^2 \cdot 4 \cdot \pi \cdot r^2 \, dr = 0.215$       Kinetic energy:  $E(\beta) - 0.215 = 0.571$

f. Calculate the probability that the particle is in the barrier.

$$1 - \int_0^2 \Psi(r, \beta)^2 \cdot 4 \cdot \pi \cdot r^2 \, dr = 0.107$$

g. Plot the wavefunction on the same graph as the potential energy.

