Approximate Quantum Mechanical Methods
Variation Method for a Particle in a 1D Ice Cream Cone

Define potential energy: \( V(x) := |x| \)

Display potential energy:

Choose trial wave function: \( \Psi(x, \beta) := \sqrt{\frac{\beta}{2}} \text{sech}(\beta x) \)

\[
E(\beta) := \int_{-\infty}^{\infty} \Psi(x, \beta) \left( -\frac{1}{2} \frac{d^2}{dx^2} \Psi(x, \beta) \right) dx + \int_{-\infty}^{\infty} V(x) \Psi(x, \beta)^2 dx
\]

Minimize the energy integral with respect to the variational parameter, \( \beta \).

\[ \beta := 0.2 \quad \text{Minimize}(E, \beta) \quad \beta = 1.276 \quad E(\beta) = 0.815 \]

Display wave function in the potential well.
Calculate the probability that the particle is in the potential barrier.

\[ 2 \int_{0}^{\infty} \Psi(x, \beta)^2 \, dx = 1 \]

Define quantum mechanical tunneling.

Tunneling occurs when a quon (a quantum mechanical particle) has probability of being in a nonclassical region. In other words, a region in which the total energy is less than the potential energy.

Calculate the probability that tunneling is occurring.

\[ |x| = 0.815 \quad \text{solve, } \{ x \rightarrow 0.815, -0.815 \} \]

\[ 2 \int_{0.815}^{\infty} \Psi(x, \beta)^2 \, dx = 0.222 \]

Calculate the kinetic and potential energy contributions to the total energy.

**Kinetic energy:**

\[ \int_{-\infty}^{\infty} \frac{1}{2} \frac{d^2}{dx^2} \Psi(x, \beta) \, dx = 0.272 \]

**Potential energy:**

\[ \int_{-\infty}^{\infty} V(x) \cdot \Psi(x, \beta)^2 \, dx = 0.543 \]