Trial Wave Functions for Various Potentials
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This is list of functions and the potentials for which they would be suitable trial wave functions in a variation method calculation.

\[
\Psi(x, \alpha) := 2^{3/2} \cdot \alpha \cdot x \cdot \exp(-\alpha \cdot x)
\]
\[
\Psi(x, \alpha) := \left( \frac{128 \cdot \alpha^3}{\pi^2} \right) \cdot x \cdot \exp(-\alpha \cdot x^2)
\]

- Particle in a gravitational field \( V(z) = mgz \) \((z = 0 \text{ to } \infty)\)
- Particle confined by a linear potential \( V(x) = ax \) \((x = 0 \text{ to } \infty)\)
- One-dimensional atoms and ions \( V(x) = -Z/x \) \((x = 0 \text{ to } \infty)\)
- Particle in semi-infinite potential well \( V(x) = \text{if} [x \leq a, 0, b] \) \((x = 0 \text{ to } \infty)\)
- Particle in semi-harmonic potential well \( V(x) = kx^2 \) \((x = 0 \text{ to } \infty)\)
- Quartic oscillator \( V(x) = bx^4 \) \((x = -\infty \text{ to } \infty)\)
- Particle in the finite one-dimensional potential well \( V(x) := \text{if} [(x \geq -1) \cdot (x \leq 1), 0, 2] \) \((x = -\infty \text{ to } \infty)\)
- 1D Hydrogen atom ground state
- Harmonic oscillator ground state
- Particle in \( V(x) = |x| \) potential well

\[
\Psi(x, \alpha) := \sqrt{\alpha} \cdot \exp(-\alpha \cdot |x|)
\]

- This wavefunction is discontinuous at \( x = 0 \), so the following calculations must be made in momentum space
- Dirac hydrogen atom \( V(x) := -\Delta(x) \)
- Harmonic oscillator ground state
- Particle in \( V(x) = |x| \) potential well
- Quartic oscillator \( V(x) = bx^4 \) \((x = -\infty \text{ to } \infty)\)

\[
\Psi(x) := \sqrt{30} \cdot x \cdot (1 - x) \quad \Gamma(x) := \sqrt{105} \cdot x \cdot (1 - x)^2 \quad \Theta(x) := \sqrt{105} \cdot x^2 \cdot (1 - x)
\]

- Particle in a one-dimensional, one-bohr box
- Particle in a slanted one-dimensional box
- Particle in a semi-infinite potential well (change 1 to variational parameter)
- Particle in a gravitational field (change 1 to variational parameter)

\[
\Phi(r, a) := (a - r) \quad \Phi(r, a) := (a - r)^2  \quad \Phi(r, a) := \frac{1}{\sqrt{2 \cdot \pi \cdot a}} \cdot \frac{\sin\left(\frac{\pi \cdot r}{a}\right)}{r}
\]

- Particle in a infinite spherical potential well of radius \( a \)
- Particle in a finite spherical potential well (treat \( a \) as a variational parameter)
\[ \Psi(r, \beta) := \left( \frac{2 \cdot \beta}{\pi} \right)^4 \cdot \exp\left(-\beta \cdot r^2\right) \]

- Particle in a finite spherical potential well
- Hydrogen atom ground state
- Helium atom ground state

\[ \Psi(r, \beta) := \sqrt{\frac{3 \cdot \beta^3}{\pi^3}} \cdot \text{sech}(\beta \cdot r) \]

- Particle in a finite potential well
- Hydrogen atom ground state
- Helium atom ground state

\[ \Psi(x, \beta) := \sqrt{\frac{\beta}{2}} \cdot \text{sech}(\beta \cdot x) \]

- Harmonic oscillator
- Quartic oscillator
- Particle in a gravitational field
- Particle in a finite potential well

\[ \Psi(\alpha, x) := \frac{\sqrt{12 \cdot \alpha^3}}{\pi} \cdot x \cdot \text{sech}(\alpha \cdot x) \]

- Particle in a semi-infinite potential well
- Particle in a gravitational field
- Particle in a linear potential well (same as above) \( V(x) = a x \) \((x = 0 \text{ to } \infty)\)
- 1D hydrogen atom or one-electron ion

Some finite potential energy wells.

\[ V(x) := \text{if}[(x \geq -1) \cdot (x \leq 1), 0, V_0] \quad V(x) := \text{if}[(x \geq -1) \cdot (x \leq 1), 0, |x| - 1] \]

\[ V(x) := \text{if}[(x \geq -1) \cdot (x \leq 1), 0, \sqrt{|x| - 1}] \]

Some semi-infinite potential energy well.

\[ V(x) := \text{if}(x \leq a, 0, b) \quad V(x) := \text{if}[(x \leq 2), 0, \frac{5}{x}] \quad V(x) := \text{if}[(x \leq 2), 0, (x - 2)] \]

\[ V(x) := \text{if}[(x \leq 2), 0, \sqrt{x - 2}] \]