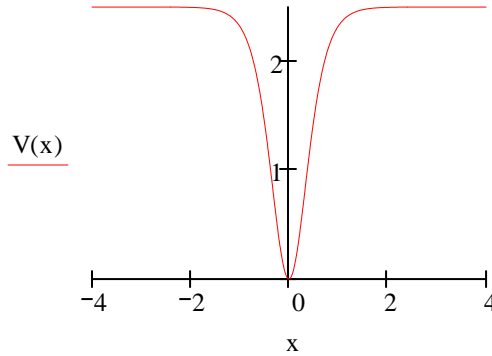


Variation Method Using the Wigner Function: The Feshbach Potential

Define potential energy: $V_0 := 2.5$ $d := 0.5$ $V(x) := V_0 \cdot \tanh\left(\frac{x}{d}\right)^2$

Display potential energy:



Choose trial wave function: $\Psi(x, \beta) := \left(\frac{2 \cdot \beta}{\pi}\right)^{\frac{1}{4}} \cdot \exp(-\beta \cdot x^2)$

Calculate the Wigner distribution function:

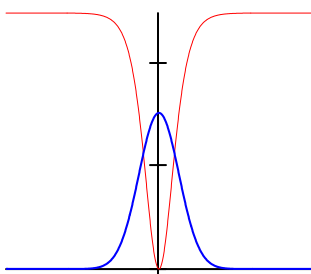
$$W(x, p, \beta) := \frac{1}{2\pi} \cdot \int_{-\infty}^{\infty} \Psi\left(x + \frac{s}{2}, \beta\right) \cdot \exp(i \cdot s \cdot p) \cdot \Psi\left(x - \frac{s}{2}, \beta\right) ds \quad \left| \begin{array}{l} \text{simplify} \\ \text{assume, } \beta > 0 \end{array} \right. \rightarrow \frac{1}{\pi} \cdot e^{\frac{-1}{2} \cdot \frac{4 \cdot \beta^2 \cdot x^2 + p^2}{\beta}}$$

Evaluate the variational integral: $E(\beta) := \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} W(x, p, \beta) \cdot \left(\frac{p^2}{2} + V(x)\right) dx dp$

Minimize the energy integral with respect to the variational parameter, β .

$$\beta := 1 \quad \beta := \text{Minimize}(E, \beta) \quad \beta = 0.913 \quad E(\beta) = 1.484$$

Calculate and display the coordinate distribution function: $P_x(x, \beta) := \int_{-\infty}^{\infty} W(x, p, \beta) dp$

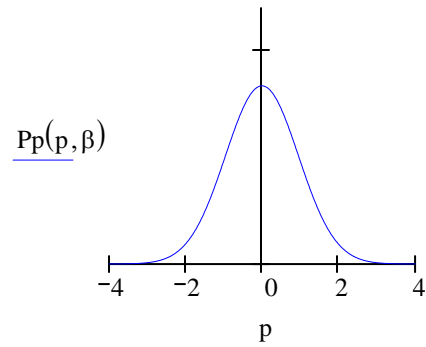


Classical turning point: $V_0 \cdot \tanh\left(\frac{x}{d}\right)^2 = 1.484 \quad \left| \begin{array}{l} \text{solve, } x \\ \text{float, } 3 \end{array} \right. \rightarrow \begin{pmatrix} -.511 \\ .511 \end{pmatrix}$

Probability that tunneling is occurring: $2 \cdot \int_{0.511}^{\infty} P_x(x, \beta) dx = 0.329$

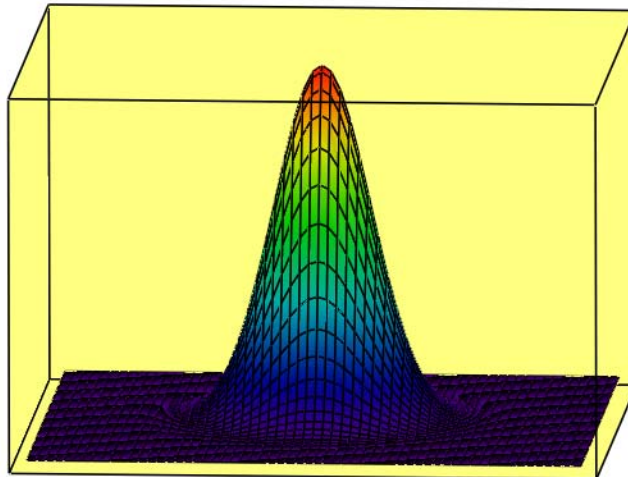
Calculate and display the momentum distribution function:

$$P_p(p, \beta) := \int_{-\infty}^{\infty} W(x, p, \beta) dx$$



Display the Wigner distribution function:

$$N := 60 \quad i := 0..N \quad x_i := -3 + \frac{6 \cdot i}{N} \quad j := 0..N \quad p_j := -5 + \frac{10 \cdot j}{N} \quad \text{Wigner}_{i,j} := W(x_i, p_j, \beta)$$



Wigner