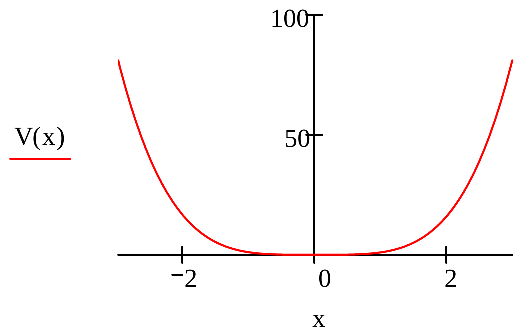
Variation Method Using the Wigner Function: The Quartic Oscillator

Define potential energy: $V(x) := x^4$

Display potential energy:



Choose trial wave function: $\Psi(x,\beta) := \left(\frac{2\cdot\beta}{\pi}\right)^{\frac{1}{4}}\cdot\exp\left(-\beta\cdot x^2\right)$

Calculate the Wigner distribution function:

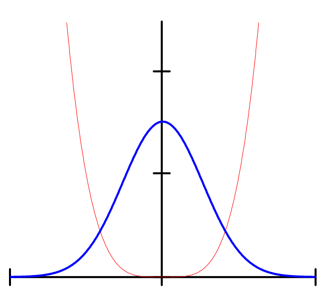
$$W(x,p,\beta) := \frac{1}{2\pi}\cdot\int_{-\infty}^{\infty}\Psi\left(x+\frac{s}{2},\beta\right)\cdot\exp(i\cdot s\cdot p)\cdot\Psi\left(x-\frac{s}{2},\beta\right)\,ds \;\Bigg|\;\begin{array}{l}\text{simplify}\\ \text{assume}, \beta > 0\end{array} \rightarrow \frac{1}{\pi}\cdot e^{\frac{-1}{2}\cdot\frac{4\cdot\beta^2\cdot x^2+p^2}{\beta}}$$

Evaluate the variational integral: $E(\beta) := \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}W(x,p,\beta)\cdot\left(\frac{p^2}{2}+V(x)\right)\,dx\,dp$

Minimize the energy integral with respect to the variational parameter, β.

$\beta := 1$ $\beta := \text{Minimize}(E,\beta)$ $\beta = 0.907$ $E(\beta) = 0.681$

Calculate and display the coordinate distribution function: $Px(x,\beta) := \int_{-\infty}^{\infty}W(x,p,\beta)\,dp$

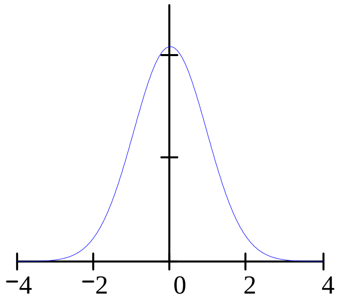


Classical turning point: $x_{cl} := 0.681^{\frac{1}{4}}$ $x_{cl} = 0.908$

Probability that tunneling is occuring: $2\cdot\int_{0.908}^{\infty}Px(x,\beta)\,dx = 0.084$

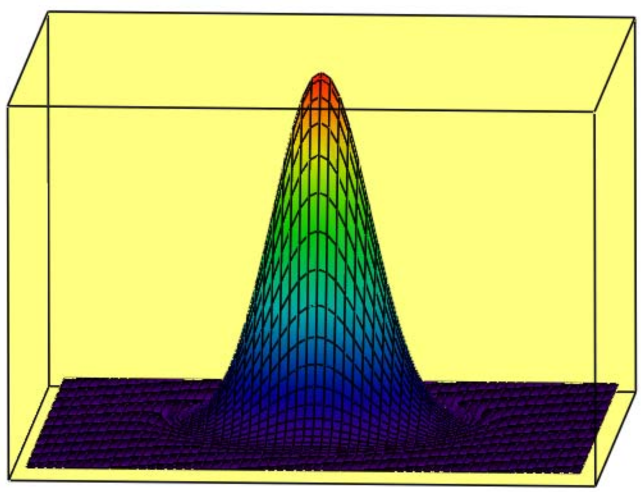
Calculate and display the momentum distribution function:

$$Pp(p, \beta) := \int_{-\infty}^{\infty} W(x, p, \beta)\, dx$$



Display the Wigner distribution function:

$N := 60$ $\quad i := 0 .. N$ $\quad x_i := -3 + \frac{6 \cdot i}{N}$ $\quad j := 0 .. N$ $\quad p_j := -5 + \frac{10 \cdot j}{N}$ $\quad Wigner_{i,j} := W(x_i, p_j, \beta)$



Wigner