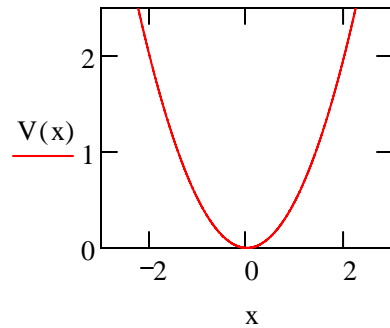


Variation Method for a Particle Traped in Harmonic Potential Well

Define potential energy: $V(x) := \frac{x^2}{2}$

Display potential energy:



Choose trial wave function: $\Psi(x, \beta) := \sqrt{\frac{\beta}{2}} \cdot \text{sech}(\beta \cdot x)$

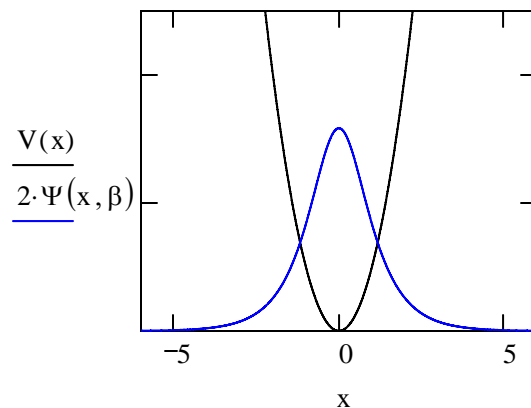
Set up variational energy integral:

$$E(\beta) := \int_{-\infty}^{\infty} \Psi(x, \beta) \cdot \left[-\frac{1}{2} \cdot \frac{d^2}{dx^2} \Psi(x, \beta) \right] dx + \int_{-\infty}^{\infty} V(x) \cdot \Psi(x, \beta)^2 dx \quad \left| \begin{array}{l} \text{assume, } \beta > 0 \\ \text{simplify} \end{array} \right. \rightarrow \frac{1}{24} \cdot \frac{4 \cdot \beta^4 + \pi^2}{\beta^2}$$

Minimize the energy integral with respect to the variational parameter, β .

$$\beta := .2 \quad \beta := \text{Minimize}(E, \beta) \quad \beta = 1.253 \quad E(\beta) = 0.524$$

Display wave function in the potential well.



Calculate the probability that the particle is in the potential barrier.

$$2 \cdot \int_0^{\infty} \Psi(x, \beta)^2 dx = 1$$

Define quantum mechanical tunneling.

Tunneling occurs when a quon (a quantum mechanical particle) has probability of being in a nonclassical region. In other words, a region in which the total energy is less than the potential energy.

Calculate the probability that tunneling is occurring.

Calculate the classical turning point.

$$\frac{x^2}{2} = 0.524 \quad \left| \begin{array}{l} \text{solve, x} \\ \text{float, 4} \end{array} \right. \rightarrow \begin{pmatrix} -1.024 \\ 1.024 \end{pmatrix}$$

$$2 \cdot \int_{1.024}^{\infty} \Psi(x, \beta)^2 dx = 0.143$$

Calculate the kinetic and potential energy contributions to the total energy.

Kinetic energy:
$$\int_{-\infty}^{\infty} \Psi(x, \beta) \cdot \left(-\frac{1}{2} \cdot \frac{d^2}{dx^2}\right) \Psi(x, \beta) dx = 0.262$$

Potential energy:
$$\int_{-\infty}^{\infty} V(x) \cdot \Psi(x, \beta)^2 dx = 0.262$$

Is the virial theorem satisfied?

Yes, for the harmonic potential the virial theorem is $T = V = E/2$.