

Is a Two-dimensional Fibonacci Array a Quasilattice?

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A two-dimensional Fibonacci lattice lacks translational periodicity but has a discrete diffraction pattern, just like a quasicrystal. However, it does not fit the definition of a quasilattice because it does not possess one of the 'forbidden' n-fold rotational symmetries (n = 5 or greater than 6) that are characteristic of quasicrystals and incompatible with translational periodicity. R. Lifshitz¹, therefore, recommends that the symmetry requirement be relaxed so that two- and three-dimensional Fibonacci lattices can have quasilattice stature.

A one-dimensional Fibonacci grid consists of a sequence of long (L) and short (S) segments such as LSLLSLSLLS... with L/S = 1.618, the golden ratio. A two-dimensional array is created by superimposing two such grids at a 90° angle and placing atomic scatterers at the vertices.

Dimension of grid: A := 10 m := 1 .. A n := 1 .. A $\tau := \frac{1 + \sqrt{5}}{2}$

Calculate the coordinates of the Fibonacci vertices in a two-dimensional lattice (see Lifshitz).

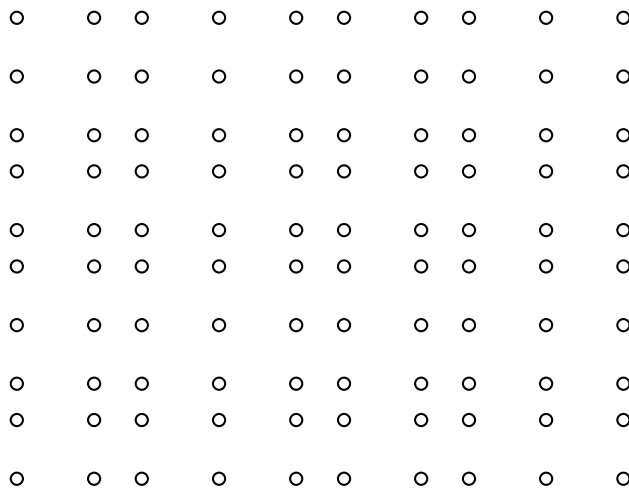
$$x_m := \text{floor}\left(\frac{m}{\tau}\right) \cdot \tau + \left(m - \text{floor}\left(\frac{m}{\tau}\right)\right) - 1 \qquad y_n := \text{floor}\left(\frac{n}{\tau}\right) \cdot \tau + \left(n - \text{floor}\left(\frac{n}{\tau}\right)\right) - 1$$

$$x^T = (0 \ 1.618 \ 2.618 \ 4.236 \ 5.854 \ 6.854 \ 8.472 \ 9.472 \ 11.09 \ 12.708)$$

$$y^T = (0 \ 1.618 \ 2.618 \ 4.236 \ 5.854 \ 6.854 \ 8.472 \ 9.472 \ 11.09 \ 12.708)$$

Display the two-dimensional Fibonacci array:

2D Fibonacci Lattice



The diffraction pattern is the Fourier transform of the spatial Fibonacci array into the momentum representation.

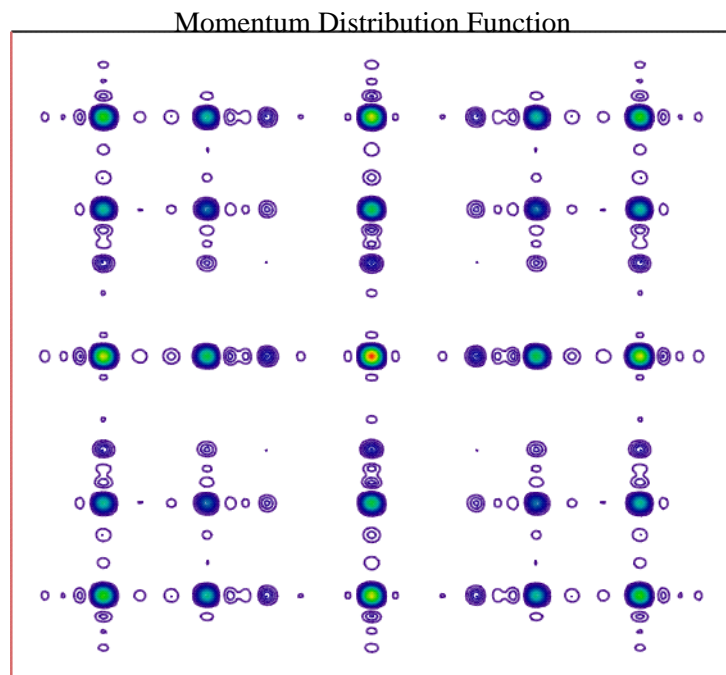
Calculate momentum-space wave function:

$$\Phi(p_x, p_y) := \frac{1}{2 \cdot \pi} \cdot \sum_{m=1}^A \exp(-i \cdot p_x \cdot x_m) \cdot \sum_{n=1}^A \exp(-i \cdot p_y \cdot y_n)$$

Display momentum-space distribution function (diffraction pattern):

$$\Delta := 10 \quad N := 200 \quad j := 1..N \quad p_{x_j} := -\Delta + \frac{2 \cdot \Delta \cdot j}{N} \quad k := 1..N \quad p_{y_k} := -\Delta + \frac{2 \cdot \Delta \cdot k}{N}$$

$$\text{DiffractionPattern}_{j,k} := \left(\left| \Phi(p_{x_j}, p_{y_k}) \right| \right)^2$$



DiffractionPattern

1. R. Lifshitz, "The square Fibonacci tiling," *Journal of Alloys and Compounds* **342**, 186-190 (2002).

