The Difference Between Fermions and Bosons

Calculate the average separation, |x₁ - x₂|, for two fermions and two bosons in a 1D box of unit length.

Fermions have antisymmetric wave functions:

\[ Ψ_f(x_1, x_2) := \frac{Ψ(x_1)Φ(x_2) - Ψ(x_2)Φ(x_1)}{\sqrt{2}} \]

The average particle separation for indistinguishable fermions:

\[ \int_0^1 \int_0^1 Ψ_f(x_1, x_2) |x_1 - x_2| Ψ_f(x_1, x_2) \, dx_1 \, dx_2 = 0.383 \]

Bosons have symmetric wave functions:

\[ Ψ_b(x_1, x_2) := \frac{Ψ(x_1)Φ(x_2) + Ψ(x_2)Φ(x_1)}{\sqrt{2}} \]

The average particle separation for indistinguishable bosons:

\[ \int_0^1 \int_0^1 Ψ_b(x_1, x_2) |x_1 - x_2| Ψ_b(x_1, x_2) \, dx_1 \, dx_2 = 0.157 \]
The particles are correlated so as to bring them closer together.

\[
N := 40 \quad i := 0..N \quad x_1 := \frac{i}{N} \quad j := 0..N \quad x_2 := \frac{j}{N} \quad \Psi_{b_{i,j}} := \Psi_b(x_1, x_2) \]

All fundamental particles (electrons, neutrons, protons, photons, etc.) are either bosons or fermions. Composite entities such as the elements also fall into these two categories. The fundamental distinction is spin: bosons have integer spin (0, 1, 2, ...) while fermions have half-integer spin (1/2, 3/2, ...).

The dramatic difference in behavior between bosons and fermions has led to a sociology of fundamental particles. Bosons are social and gregarious, while fermions are antisocial and aloof.