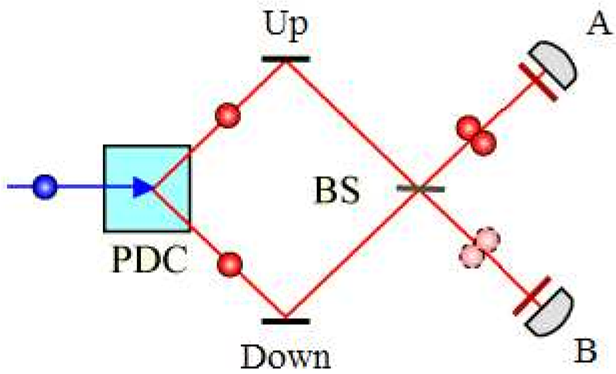
Two-Particle Interference for Bosons and Fermions



A parametric down converter, PDC, transforms an incident photon into two lower energy photons. One photon takes the upper path and the other the lower path or vice versa. The principles of quantum mechanics require that the wave function for this event be written as the following entangled superposition.

Entangled superposition for **bosons**: $\frac{1}{\sqrt{2}}\cdot\left(U_1\cdot D_2 + D_1\cdot U_2\right)$

At the beam splitter, BS, the probability amplitude for transmission is $\frac{1}{\sqrt{2}}$ and the probability amplitude for reflection is $\frac{i}{\sqrt{2}}$. Therefore, for the four possible arrivals at the detectors we have,

$$U_1 = \frac{1}{\sqrt{2}}\cdot\left(i\cdot A_1 + B_1\right) \qquad D_1 = \frac{1}{\sqrt{2}}\cdot\left(A_1 + i\cdot B_1\right) \qquad U_2 = \frac{1}{\sqrt{2}}\cdot\left(i\cdot A_2 + B_2\right) \qquad D_2 = \frac{1}{\sqrt{2}}\cdot\left(A_2 + i\cdot B_2\right)$$

Bosons are always observed at the same detector.

$$\frac{1}{\sqrt{2}}\cdot\left[\frac{1}{\sqrt{2}}\cdot\left(i\cdot A_1 + B_1\right)\cdot\frac{1}{\sqrt{2}}\cdot\left(A_2 + i\cdot B_2\right) + \frac{1}{\sqrt{2}}\cdot\left(A_1 + i\cdot B_1\right)\cdot\frac{1}{\sqrt{2}}\cdot\left(i\cdot A_2 + B_2\right)\right] \text{ simplify } \rightarrow \frac{1}{2}\cdot i\cdot 2^{\frac{1}{2}}\cdot\left(A_1\cdot A_2 + B_1\cdot B_2\right)$$

Entangled superposition for **fermions**:

$$\frac{1}{\sqrt{2}}\cdot\left(U_1\cdot D_2 - D_1\cdot U_2\right)$$

Fermions are never observed at the same detector.

$$\frac{1}{\sqrt{2}}\cdot\left[\frac{1}{\sqrt{2}}\cdot\left(i\cdot A_1 + B_1\right)\cdot\frac{1}{\sqrt{2}}\cdot\left(A_2 + i\cdot B_2\right) - \frac{1}{\sqrt{2}}\cdot\left(A_1 + i\cdot B_1\right)\cdot\frac{1}{\sqrt{2}}\cdot\left(i\cdot A_2 + B_2\right)\right] \text{ simplify } \rightarrow \frac{-1}{2}\cdot 2^{\frac{1}{2}}\cdot\left(A_1\cdot B_2 - B_1\cdot A_2\right)$$

In summary, the sociology of bosons and fermions can be briefly stated: bosons are gregarious and enjoy company; fermions are antisocial and prefer solitude.

The calculations can also be carried out in the following method using Mathcad's live Symbolic processor.

$$\text{Boson} = \frac{1}{\sqrt{2}}\cdot\left(U_1\cdot D_2 + D_1\cdot U_2\right) \left| \begin{aligned} &\text{substitute}, U_1 = \frac{1}{\sqrt{2}}\cdot\left(i\cdot A_1 + B_1\right) \\ &\text{substitute}, U_2 = \frac{1}{\sqrt{2}}\cdot\left(i\cdot A_2 + B_2\right) \\ &\text{substitute}, D_1 = \frac{1}{\sqrt{2}}\cdot\left(A_1 + i\cdot B_1\right) \\ &\text{substitute}, D_2 = \frac{1}{\sqrt{2}}\cdot\left(A_2 + i\cdot B_2\right) \\ &\text{simplify} \end{aligned} \right. \rightarrow \text{Boson} = \frac{1}{2}\cdot i\cdot 2^{\frac{1}{2}}\cdot\left(A_1\cdot A_2 + B_1\cdot B_2\right)$$

$$\text{Fermion} = \frac{1}{\sqrt{2}}\cdot\left(U_1\cdot D_2 - D_1\cdot U_2\right) \left| \begin{aligned} &\text{substitute}, U_1 = \frac{1}{\sqrt{2}}\cdot\left(i\cdot A_1 + B_1\right) \\ &\text{substitute}, U_2 = \frac{1}{\sqrt{2}}\cdot\left(i\cdot A_2 + B_2\right) \\ &\text{substitute}, D_1 = \frac{1}{\sqrt{2}}\cdot\left(A_1 + i\cdot B_1\right) \\ &\text{substitute}, D_2 = \frac{1}{\sqrt{2}}\cdot\left(A_2 + i\cdot B_2\right) \\ &\text{simplify} \end{aligned} \right. \rightarrow \text{Fermion} = \frac{-1}{2}\cdot 2^{\frac{1}{2}}\cdot\left(A_1\cdot B_2 - B_1\cdot A_2\right)$$