Numerical Solutions for Schroedinger's Equation

The Quantized Bouncing Particle

Integration limi: \( z_{\text{max}} := 3 \)
Mass: \( m := 2 \)
Acceleration due to gravity: \( g := 1 \)

The first 10 roots of the Airy function are as follows:
\[
\begin{align*}
\alpha_1 &:= 2.33810 \\
\alpha_2 &:= 4.08794 \\
\alpha_3 &:= 5.52055 \\
\alpha_4 &:= 6.78670 \\
\alpha_5 &:= 7.94413 \\
\alpha_6 &:= 8.02265 \\
\alpha_7 &:= 10.04017 \\
\alpha_8 &:= 11.00852 \\
\alpha_9 &:= 11.93601 \\
\alpha_{10} &:= 12.82877
\end{align*}
\]

Calculate energy analytically by selecting appropriate Airy function root.

\[
i := 1 \quad E := \left( \frac{m \cdot g}{2} \right) \cdot \alpha_i \quad E = 2.338
\]

Generate the associated wavefunction numerically:

Potential energy: \( V(z) := m \cdot g \cdot z \)

Given
\[
\frac{-1}{2 \cdot m} \frac{d^2}{dz^2} \Psi(z) + V(z) \cdot \Psi(z) = E \cdot \Psi(z)
\]

\[
\Psi(0.0) = 0.0 \quad \Psi'(0.0) = 0.1
\]

\[
\Psi := \text{Odesolve}(z, z_{\text{max}})
\]

Normalize wavefunction:

\[
\Psi(z) := \frac{\Psi(z)}{\sqrt{\int_0^{z_{\text{max}}}{\Psi(z)^2 \, dz}}}
\]