Quantum Entanglement Leads to Nonclassical Correlations

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A source emits the following six cubit state, with cubits 1,3 and 5 going to Alice, and cubits 2, 4 and 6 going to Bob. This example is due to P. K. Aravind and available at **arXiv:quant-ph/0701031v1**.

$$|\Psi\rangle = \frac{1}{\sqrt{2}}\left(|\alpha\rangle_1|\alpha\rangle_2 + |\beta\rangle_1|\beta\rangle_2\right) \otimes \frac{1}{\sqrt{2}}\left(|\alpha\rangle_3|\alpha\rangle_4 + |\beta\rangle_3|\beta\rangle_4\right) \otimes \frac{1}{\sqrt{2}}\left(|\alpha\rangle_5|\alpha\rangle_6 + |\beta\rangle_5|\beta\rangle_6\right)$$

$$= \frac{1}{2\sqrt{2}}\begin{bmatrix} |\alpha\rangle_1|\alpha\rangle_2|\alpha\rangle_3|\alpha\rangle_4|\alpha\rangle_5|\alpha\rangle_6 + |\alpha\rangle_1|\alpha\rangle_2|\beta\rangle_3|\beta\rangle_4|\alpha\rangle_5|\alpha\rangle_6 + |\beta\rangle_1|\beta\rangle_2|\alpha\rangle_3|\alpha\rangle_4|\alpha\rangle_5|\alpha\rangle_6 + |\beta\rangle_1|\beta\rangle_2|\beta\rangle_3|\beta\rangle_4|\alpha\rangle_5|\alpha\rangle_6 \\ + |\alpha\rangle_1|\alpha\rangle_2|\alpha\rangle_3|\alpha\rangle_4|\beta\rangle_5|\beta\rangle_6 + |\alpha\rangle_1|\alpha\rangle_2|\beta\rangle_3|\beta\rangle_4|\beta\rangle_5|\beta\rangle_6 + |\beta\rangle_1|\beta\rangle_2|\alpha\rangle_3|\alpha\rangle_4|\beta\rangle_5|\beta\rangle_6 + |\beta\rangle_1|\beta\rangle_2|\beta\rangle_3|\beta\rangle_4|\beta\rangle_5|\beta\rangle_6 \end{bmatrix}$$

The vectors required to form this initial state function in tensor format are:

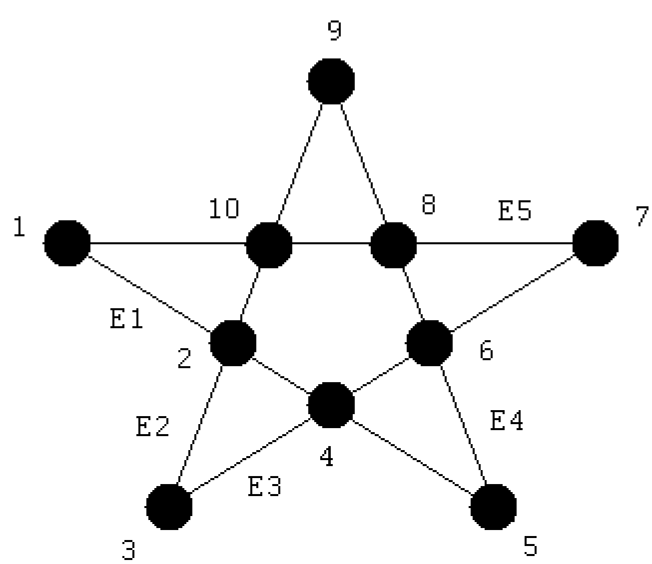
$$\alpha := \begin{pmatrix} 1 \\ 0 \end{pmatrix} \qquad \beta := \begin{pmatrix} 0 \\ 1 \end{pmatrix} \qquad n := \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

The state is formed as follows, but see the Appendix for an alternative method.

$$\alpha n := \text{augment}(\alpha, n) \qquad \beta n := \text{augment}(\beta, n)$$

$$\Psi := \frac{1}{2\cdot\sqrt{2}}\cdot\left(\text{submatrix}\begin{pmatrix} \text{kronecker}(\alpha n, \text{kronecker}(\alpha n, \text{kronecker}(\alpha n, \text{kronecker}(\alpha n, \text{kronecker}(\alpha n, \alpha n))))) \ldots \\ + \text{kronecker}(\alpha n, \text{kronecker}(\alpha n, \text{kronecker}(\beta n, \text{kronecker}(\beta n, \text{kronecker}(\alpha n, \alpha n))))) \ldots \\ + \text{kronecker}(\beta n, \text{kronecker}(\beta n, \text{kronecker}(\alpha n, \text{kronecker}(\alpha n, \text{kronecker}(\alpha n, \alpha n))))) \ldots \\ + \text{kronecker}(\beta n, \text{kronecker}(\beta n, \text{kronecker}(\beta n, \text{kronecker}(\beta n, \text{kronecker}(\alpha n, \alpha n))))) \ldots \\ + \text{kronecker}(\alpha n, \text{kronecker}(\alpha n, \text{kronecker}(\alpha n, \text{kronecker}(\alpha n, \text{kronecker}(\beta n, \beta n))))) \ldots \\ + \text{kronecker}(\alpha n, \text{kronecker}(\alpha n, \text{kronecker}(\beta n, \text{kronecker}(\beta n, \text{kronecker}(\beta n, \beta n))))) \ldots \\ + \text{kronecker}(\beta n, \text{kronecker}(\beta n, \text{kronecker}(\alpha n, \text{kronecker}(\alpha n, \text{kronecker}(\beta n, \beta n))))) \ldots \\ + \text{kronecker}(\beta n, \text{kronecker}(\beta n, \text{kronecker}(\beta n, \text{kronecker}(\beta n, \text{kronecker}(\beta n, \beta n))))) \end{pmatrix}, 1, 64, 1, 1\right)$$

The following pentagram describes a measurement protocol followed by Alice and Bob.



Alice and Bob independently and randomly select one of five measurement protocols (E1, E2, E3, E4 and E5) shown on the edges of the pentagram above six possible settings each time the source emits the entangled particles, and record the result (+1 or -1) for each vertex. After a statistically meaningful number of events they compare their results.

Each vertex represents a measurement sequence given in the following table.

$$\begin{pmatrix} \text{Operator} & \text{Alice} & \text{Bob} \\ 1 & A(-\sigma_z, I, \sigma_z, I, \sigma_z, I) & B(I, -\sigma_z, I, \sigma_z, I, \sigma_z) \\ 2 & A(\sigma_z, I, I, I, I, I) & B(I, \sigma_z, I, I, I, I) \\ 3 & A(I, I, \sigma_x, I, I, I) & B(I, I, I, \sigma_x, I, I) \\ 4 & A(I, I, I, I, \sigma_z, I) & B(I, I, I, I, I, \sigma_z) \\ 5 & A(I, I, \sigma_z, I, I, I) & B(I, I, I, \sigma_z, I, I) \\ 6 & A(\sigma_x, I, I, I, I, I) & B(I, \sigma_x, I, I, I, I) \\ 7 & A(-\sigma_x, I, \sigma_x, I, \sigma_z, I) & B(I, -\sigma_x, I, \sigma_x, I, \sigma_z) \\ 8 & A(-\sigma_x, I, \sigma_z, I, \sigma_x, I) & B(I, -\sigma_x, I, \sigma_z, I, \sigma_x) \\ 9 & A(I, I, I, I, \sigma_x, I) & B(I, I, I, I, I, \sigma_x) \\ 10 & A(-\sigma_z, I, \sigma_x, I, \sigma_x, I) & B(I, -\sigma_z, I, \sigma_x, I, \sigma_x) \end{pmatrix}$$

The measurement operators required are:

$$I := \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \qquad \sigma_x := \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \qquad \sigma_z := \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \qquad \text{eigenvals}(\sigma_x) = \begin{pmatrix} -1 \\ 1 \end{pmatrix} \qquad \text{eigenvals}(\sigma_z) = \begin{pmatrix} -1 \\ 1 \end{pmatrix}$$

$$A(a,b,c,d,e,f) := \text{kronecker}(a,\text{kronecker}(b,\text{kronecker}(c,\text{kronecker}(d,\text{kronecker}(e,f)))))$$

$$B(a,b,c,d,e,f) := \text{kronecker}(a,\text{kronecker}(b,\text{kronecker}(c,\text{kronecker}(d,\text{kronecker}(e,f)))))$$

The eigenvalues of the Pauli operators are +/- 1. The first thing to note is that the individual vertices (measurement site) flash +1 and -1 one randomly giving expectation values of zero at each vertex.

$$\begin{pmatrix} \Psi^T \cdot A(-\sigma_z, I, \sigma_z, I, \sigma_z, I) \cdot \Psi & \Psi^T \cdot B(I, -\sigma_z, I, \sigma_z, I, \sigma_z) \cdot \Psi \\ \Psi^T \cdot A(\sigma_z, I, I, I, I, I) \cdot \Psi & \Psi^T \cdot B(I, \sigma_z, I, I, I, I) \cdot \Psi \\ \Psi^T \cdot A(I, I, \sigma_x, I, I, I) \cdot \Psi & \Psi^T \cdot B(I, I, I, \sigma_x, I, I) \cdot \Psi \\ \Psi^T \cdot A(I, I, I, I, \sigma_z, I) \cdot \Psi & \Psi^T \cdot B(I, I, I, I, I, \sigma_z) \cdot \Psi \\ \Psi^T \cdot A(I, I, \sigma_z, I, I, I) \cdot \Psi & \Psi^T \cdot B(I, I, I, \sigma_z, I, I) \cdot \Psi \\ \Psi^T \cdot A(\sigma_x, I, I, I, I, I) \cdot \Psi & \Psi^T \cdot B(I, \sigma_x, I, I, I, I) \cdot \Psi \\ \Psi^T \cdot A(-\sigma_x, I, \sigma_x, I, \sigma_z, I) \cdot \Psi & \Psi^T \cdot B(I, -\sigma_x, I, \sigma_x, I, \sigma_z) \cdot \Psi \\ \Psi^T \cdot A(-\sigma_x, I, \sigma_z, I, \sigma_x, I) \cdot \Psi & \Psi^T \cdot B(I, -\sigma_x, I, \sigma_z, I, \sigma_x) \cdot \Psi \\ \Psi^T \cdot A(I, I, I, I, \sigma_x, I) \cdot \Psi & \Psi^T \cdot B(I, I, I, I, I, \sigma_x) \cdot \Psi \\ \Psi^T \cdot A(-\sigma_z, I, \sigma_x, I, \sigma_x, I) \cdot \Psi & \Psi^T \cdot B(I, -\sigma_z, I, \sigma_x, I, \sigma_x) \cdot \Psi \end{pmatrix} = \begin{pmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{pmatrix}$$

However, if Alice and Bob make the same measurement protocol they always get the same eigenvalue, as is shown by the calculations below.

$$\begin{pmatrix} \Psi^T\cdot A(-\sigma_z, I, \sigma_z, I, \sigma_z, I)\cdot B(I, -\sigma_z, I, \sigma_z, I, \sigma_z)\cdot\Psi \\ \Psi^T\cdot A(\sigma_z, I, I, I, I, I)\cdot B(I, \sigma_z, I, I, I, I)\cdot\Psi \\ \Psi^T\cdot A(I, I, \sigma_x, I, I, I)\cdot B(I, I, I, \sigma_x, I, I)\cdot\Psi \\ \Psi^T\cdot A(I, I, I, I, \sigma_z, I)\cdot B(I, I, I, I, I, \sigma_z)\cdot\Psi \\ \Psi^T\cdot A(I, I, \sigma_z, I, I, I)\cdot B(I, I, I, \sigma_z, I, I)\cdot\Psi \\ \Psi^T\cdot A(\sigma_x, I, I, I, I, I)\cdot B(I, \sigma_x, I, I, I, I)\cdot\Psi \\ \Psi^T\cdot A(-\sigma_x, I, \sigma_x, I, \sigma_z, I)\cdot B(I, -\sigma_x, I, \sigma_x, I, \sigma_z)\cdot\Psi \\ \Psi^T\cdot A(-\sigma_x, I, \sigma_z, I, \sigma_x, I)\cdot B(I, -\sigma_x, I, \sigma_z, I, \sigma_x)\cdot\Psi \\ \Psi^T\cdot A(I, I, I, I, \sigma_x, I)\cdot B(I, I, I, I, I, \sigma_x)\cdot\Psi \\ \Psi^T\cdot A(-\sigma_z, I, \sigma_x, I, \sigma_x, I)\cdot B(I, -\sigma_z, I, \sigma_x, I, \sigma_x)\cdot\Psi \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \end{pmatrix}$$

This result is somewhat surprising given the result immediately above. It suggest (at this point) that the particles involved in this experiment carry instruction sets telling the detectors how to operate, and that Alice and Bob's detectors receive the same set of instructions.

The edges (E1 through E5) shown on the pentagram identify a sequence of four commuting operations. The net eigenvalues for these sequences of measurements for Alice and Bob are now calculated.

$$\begin{pmatrix} \Psi^T\cdot A(-\sigma_z, I, \sigma_z, I, \sigma_z, I)\cdot A(\sigma_z, I, I, I, I, I)\cdot A(I, I, I, I, \sigma_z, I)\cdot A(I, I, \sigma_z, I, I, I)\cdot\Psi \\ \Psi^T\cdot A(I, I, \sigma_x, I, I, I)\cdot A(I, I, I, I, \sigma_z, I)\cdot A(\sigma_x, I, I, I, I, I)\cdot A(-\sigma_x, I, \sigma_x, I, \sigma_z, I)\cdot\Psi \\ \Psi^T\cdot A(I, I, \sigma_x, I, I, I)\cdot A(\sigma_z, I, I, I, I, I)\cdot A(-\sigma_z, I, \sigma_x, I, \sigma_x, I)\cdot A(I, I, I, I, \sigma_x, I)\cdot\Psi \\ \Psi^T\cdot A(I, I, \sigma_z, I, I, I)\cdot A(\sigma_x, I, I, I, I, I)\cdot A(-\sigma_x, I, \sigma_z, I, \sigma_x, I)\cdot A(I, I, I, I, \sigma_x, I)\cdot\Psi \\ \Psi^T\cdot A(-\sigma_z, I, \sigma_z, I, \sigma_z, I)\cdot A(-\sigma_z, I, \sigma_x, I, \sigma_x, I)\cdot A(-\sigma_x, I, \sigma_z, I, \sigma_x, I)\cdot A(-\sigma_x, I, \sigma_x, I, \sigma_z, I)\cdot\Psi \end{pmatrix} = \begin{pmatrix} -1 \\ -1 \\ -1 \\ -1 \\ -1 \end{pmatrix}$$

$$\begin{pmatrix} \Psi^T\cdot B(I, -\sigma_z, I, \sigma_z, I, \sigma_z)\cdot B(I, \sigma_z, I, I, I, I)\cdot B(I, I, I, I, I, \sigma_z)\cdot B(I, I, I, \sigma_z, I, I)\cdot\Psi \\ \Psi^T\cdot B(I, I, I, \sigma_x, I, I)\cdot B(I, I, I, I, I, \sigma_z)\cdot B(I, \sigma_x, I, I, I, I)\cdot B(I, -\sigma_x, I, \sigma_x, I, \sigma_z)\cdot\Psi \\ \Psi^T\cdot B(I, I, I, \sigma_x, I, I)\cdot B(I, \sigma_z, I, I, I, I)\cdot B(I, -\sigma_z, I, \sigma_x, I, \sigma_x)\cdot B(I, I, I, I, I, \sigma_x)\cdot\Psi \\ \Psi^T\cdot B(I, I, I, \sigma_z, I, I)\cdot B(I, \sigma_x, I, I, I, I)\cdot B(I, -\sigma_x, I, \sigma_z, I, \sigma_x)\cdot B(I, I, I, I, I, \sigma_x)\cdot\Psi \\ \Psi^T\cdot B(I, -\sigma_z, I, \sigma_z, I, \sigma_z)\cdot B(I, -\sigma_z, I, \sigma_x, I, \sigma_x)\cdot B(I, -\sigma_x, I, \sigma_z, I, \sigma_x)\cdot B(I, -\sigma_x, I, \sigma_x, I, \sigma_z)\cdot\Psi \end{pmatrix} = \begin{pmatrix} -1 \\ -1 \\ -1 \\ -1 \\ -1 \end{pmatrix}$$

We now have the problem of reconciling these results with those immediately prior. How is it possible to assign +1s and -1s to the measurement vertices such that they satisfy the results immediately above. The answer is that it is impossible. The ideas of instruction sets and elements of reality are not capable of explaining these results.

Appendix

An alternative method of expressing the initial state in tensor format:

$$|\Psi\rangle = \frac{1}{\sqrt{2}}\left(|\alpha\rangle_1|\alpha\rangle_2 + |\beta\rangle_1|\beta\rangle_2\right) \otimes \frac{1}{\sqrt{2}}\left(|\alpha\rangle_3|\alpha\rangle_4 + |\beta\rangle_3|\beta\rangle_4\right) \otimes \frac{1}{\sqrt{2}}\left(|\alpha\rangle_5|\alpha\rangle_6 + |\beta\rangle_5|\beta\rangle_6\right)$$

$$= \frac{1}{\sqrt{2}}\left[\begin{pmatrix}1\\0\end{pmatrix}\otimes\begin{pmatrix}1\\0\end{pmatrix}+\begin{pmatrix}0\\1\end{pmatrix}\otimes\begin{pmatrix}0\\1\end{pmatrix}\right]\otimes\frac{1}{\sqrt{2}}\left[\begin{pmatrix}1\\0\end{pmatrix}\otimes\begin{pmatrix}1\\0\end{pmatrix}+\begin{pmatrix}0\\1\end{pmatrix}\otimes\begin{pmatrix}0\\1\end{pmatrix}\right]\frac{1}{\sqrt{2}}\left[\begin{pmatrix}1\\0\end{pmatrix}\otimes\begin{pmatrix}1\\0\end{pmatrix}+\begin{pmatrix}0\\1\end{pmatrix}\otimes\begin{pmatrix}0\\1\end{pmatrix}\right] = \frac{1}{\sqrt{2}}\begin{pmatrix}1\\0\\0\\1\end{pmatrix}\otimes\frac{1}{\sqrt{2}}\begin{pmatrix}1\\0\\0\\1\end{pmatrix}\otimes\frac{1}{\sqrt{2}}\begin{pmatrix}1\\0\\0\\1\end{pmatrix}$$

Place the three vectors in the left-hand columns of three 4x4 matrices, carry out their tensor multiplication with Mathcad's **kronecker** command, and extract Ψ from the left-hand column of the final matrix using the **submatrix** command.

$$\Psi := \frac{1}{2\cdot\sqrt{2}}\cdot\text{submatrix}\left[\text{kronecker}\left[\begin{pmatrix}1&0&0&0\\0&0&0&0\\0&0&0&0\\1&0&0&0\end{pmatrix},\text{kronecker}\left[\begin{pmatrix}1&0&0&0\\0&0&0&0\\0&0&0&0\\1&0&0&0\end{pmatrix},\begin{pmatrix}1&0&0&0\\0&0&0&0\\0&0&0&0\\1&0&0&0\end{pmatrix}\right]\right],1,64,1,1\right]$$

All the calculations carried out above can also be done having the trace function operate on the product of the density matrix for Ψ and the matrices representing the measurement operators.

Form the density matrix for Ψ. $\rho_\Psi := \Psi\cdot\Psi^T$

$$\begin{pmatrix}
\text{tr}(\rho_\Psi\cdot A(-\sigma_z,I,\sigma_z,I,\sigma_z,I)) & \text{tr}(\rho_\Psi\cdot B(I,-\sigma_z,I,\sigma_z,I,\sigma_z))\\
\text{tr}(\rho_\Psi\cdot A(\sigma_z,I,I,I,I,I)) & \text{tr}(\rho_\Psi\cdot B(I,\sigma_z,I,I,I,I))\\
\text{tr}(\rho_\Psi\cdot A(I,I,\sigma_x,I,I,I)) & \text{tr}(\rho_\Psi\cdot B(I,I,I,\sigma_x,I,I))\\
\text{tr}(\rho_\Psi\cdot A(I,I,I,I,\sigma_z,I)) & \text{tr}(\rho_\Psi\cdot B(I,I,I,I,I,\sigma_z))\\
\text{tr}(\rho_\Psi\cdot A(I,I,\sigma_z,I,I,I)) & \text{tr}(\rho_\Psi\cdot B(I,I,I,\sigma_z,I,I))\\
\text{tr}(\rho_\Psi\cdot A(\sigma_x,I,I,I,I,I)) & \text{tr}(\rho_\Psi\cdot B(I,\sigma_x,I,I,I,I))\\
\text{tr}(\rho_\Psi\cdot A(-\sigma_x,I,\sigma_x,I,\sigma_z,I)) & \text{tr}(\rho_\Psi\cdot B(I,-\sigma_x,I,\sigma_x,I,\sigma_z))\\
\text{tr}(\rho_\Psi\cdot A(-\sigma_x,I,\sigma_z,I,\sigma_x,I)) & \text{tr}(\rho_\Psi\cdot B(I,-\sigma_x,I,\sigma_z,I,\sigma_x))\\
\text{tr}(\rho_\Psi\cdot A(I,I,I,I,\sigma_x,I)) & \text{tr}(\rho_\Psi\cdot B(I,I,I,I,I,\sigma_x))\\
\text{tr}(\rho_\Psi\cdot A(-\sigma_z,I,\sigma_x,I,\sigma_x,I)) & \text{tr}(\rho_\Psi\cdot B(I,-\sigma_z,I,\sigma_x,I,\sigma_x))
\end{pmatrix} = \begin{pmatrix}0&0\\0&0\\0&0\\0&0\\0&0\\0&0\\0&0\\0&0\\0&0\\0&0\end{pmatrix}$$

$$\begin{pmatrix} \mathrm{tr}\left(\rho_{\Psi}\cdot A\left(-\sigma_z, I, \sigma_z, I, \sigma_z, I\right)\cdot B\left(I, -\sigma_z, I, \sigma_z, I, \sigma_z\right)\right) \\ \mathrm{tr}\left(\rho_{\Psi}\cdot A\left(\sigma_z, I, I, I, I, I\right)\cdot B\left(I, \sigma_z, I, I, I, I\right)\right) \\ \mathrm{tr}\left(\rho_{\Psi}\cdot A\left(I, I, \sigma_x, I, I, I\right)\cdot B\left(I, I, I, \sigma_x, I, I\right)\right) \\ \mathrm{tr}\left(\rho_{\Psi}\cdot A\left(I, I, I, I, \sigma_z, I\right)\cdot B\left(I, I, I, I, I, \sigma_z\right)\right) \\ \mathrm{tr}\left(\rho_{\Psi}\cdot A\left(I, I, \sigma_z, I, I, I\right)\cdot B\left(I, I, I, \sigma_z, I, I\right)\right) \\ \mathrm{tr}\left(\rho_{\Psi}\cdot A\left(\sigma_x, I, I, I, I, I\right)\cdot B\left(I, \sigma_x, I, I, I, I\right)\right) \\ \mathrm{tr}\left(\rho_{\Psi}\cdot A\left(-\sigma_x, I, \sigma_x, I, \sigma_z, I\right)\cdot B\left(I, -\sigma_x, I, \sigma_x, I, \sigma_z\right)\right) \\ \mathrm{tr}\left(\rho_{\Psi}\cdot A\left(-\sigma_x, I, \sigma_z, I, \sigma_x, I\right)\cdot B\left(I, -\sigma_x, I, \sigma_z, I, \sigma_x\right)\right) \\ \mathrm{tr}\left(\rho_{\Psi}\cdot A\left(I, I, I, I, \sigma_x, I\right)\cdot B\left(I, I, I, I, I, \sigma_x\right)\right) \\ \mathrm{tr}\left(\rho_{\Psi}\cdot A\left(-\sigma_z, I, \sigma_x, I, \sigma_x, I\right)\cdot B\left(I, -\sigma_z, I, \sigma_x, I, \sigma_x\right)\right) \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \end{pmatrix}$$

$$\begin{pmatrix} \mathrm{tr}\left(\rho_{\Psi}\cdot A\left(-\sigma_z, I, \sigma_z, I, \sigma_z, I\right)\cdot A\left(\sigma_z, I, I, I, I, I\right)\cdot A\left(I, I, I, I, \sigma_z, I\right)\cdot A\left(I, I, \sigma_z, I, I, I\right)\right) \\ \mathrm{tr}\left(\rho_{\Psi}\cdot A\left(I, I, \sigma_x, I, I, I\right)\cdot A\left(I, I, I, I, \sigma_z, I\right)\cdot A\left(\sigma_x, I, I, I, I, I\right)\cdot A\left(-\sigma_x, I, \sigma_x, I, \sigma_z, I\right)\right) \\ \mathrm{tr}\left(\rho_{\Psi}\cdot A\left(I, I, \sigma_x, I, I, I\right)\cdot A\left(\sigma_z, I, I, I, I, I\right)\cdot A\left(-\sigma_z, I, \sigma_x, I, \sigma_x, I\right)\cdot A\left(I, I, I, I, \sigma_x, I\right)\right) \\ \mathrm{tr}\left(\rho_{\Psi}\cdot A\left(I, I, \sigma_z, I, I, I\right)\cdot A\left(\sigma_x, I, I, I, I, I\right)\cdot A\left(-\sigma_x, I, \sigma_z, I, \sigma_x, I\right)\cdot A\left(I, I, I, I, \sigma_x, I\right)\right) \\ \mathrm{tr}\left(\rho_{\Psi}\cdot A\left(-\sigma_z, I, \sigma_z, I, \sigma_z, I\right)\cdot A\left(-\sigma_z, I, \sigma_x, I, \sigma_x, I\right)\cdot A\left(-\sigma_x, I, \sigma_z, I, \sigma_x, I\right)\cdot A\left(-\sigma_x, I, \sigma_x, I, \sigma_z, I\right)\right) \end{pmatrix} = \begin{pmatrix} -1 \\ -1 \\ -1 \\ -1 \\ -1 \end{pmatrix}$$

$$\begin{pmatrix} \mathrm{tr}\left(\rho_{\Psi}\cdot B\left(I, -\sigma_z, I, \sigma_z, I, \sigma_z\right)\cdot B\left(I, \sigma_z, I, I, I, I\right)\cdot B\left(I, I, I, I, I, \sigma_z\right)\cdot B\left(I, I, I, \sigma_z, I, I\right)\right) \\ \mathrm{tr}\left(\rho_{\Psi}\cdot B\left(I, I, I, \sigma_x, I, I\right)\cdot B\left(I, I, I, I, I, \sigma_z\right)\cdot B\left(I, \sigma_x, I, I, I, I\right)\cdot B\left(I, -\sigma_x, I, \sigma_x, I, \sigma_z\right)\right) \\ \mathrm{tr}\left(\rho_{\Psi}\cdot B\left(I, I, I, \sigma_x, I, I\right)\cdot B\left(I, \sigma_z, I, I, I, I\right)\cdot B\left(I, -\sigma_z, I, \sigma_x, I, \sigma_x\right)\cdot B\left(I, I, I, I, I, \sigma_x\right)\right) \\ \mathrm{tr}\left(\rho_{\Psi}\cdot B\left(I, I, I, \sigma_z, I, I\right)\cdot B\left(I, \sigma_x, I, I, I, I\right)\cdot B\left(I, -\sigma_x, I, \sigma_z, I, \sigma_x\right)\cdot B\left(I, I, I, I, I, \sigma_x\right)\right) \\ \mathrm{tr}\left(\rho_{\Psi}\cdot B\left(I, -\sigma_z, I, \sigma_z, I, \sigma_z\right)\cdot B\left(I, -\sigma_z, I, \sigma_x, I, \sigma_x\right)\cdot B\left(I, -\sigma_x, I, \sigma_z, I, \sigma_x\right)\cdot B\left(I, -\sigma_x, I, \sigma_x, I, \sigma_z\right)\right) \end{pmatrix} = \begin{pmatrix} -1 \\ -1 \\ -1 \\ -1 \\ -1 \end{pmatrix}$$