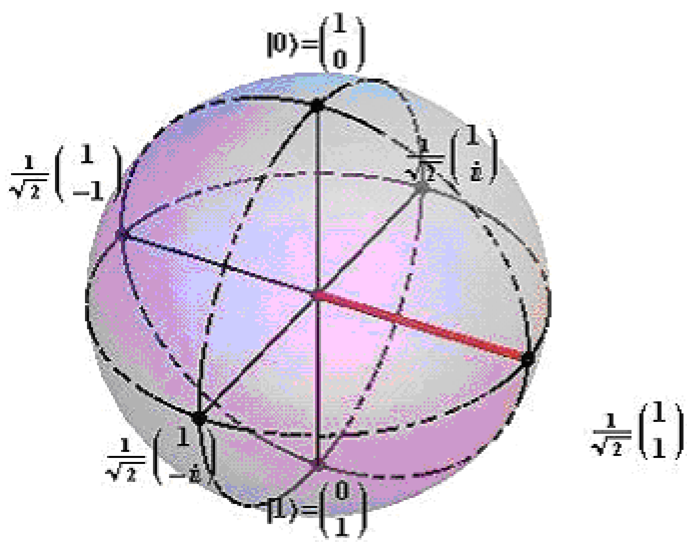
Bloch Sphere

The eigenfunctions of the Pauli spin matrices

$$\sigma_z := \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \qquad \sigma_x := \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \qquad \sigma_y := \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$$

are presented mathematically and shown on the Bloch sphere below. The X_u state is highlighted.

$$Z_u := \begin{pmatrix} 1 \\ 0 \end{pmatrix} \quad Z_d := \begin{pmatrix} 0 \\ 1 \end{pmatrix} \quad X_u := \frac{1}{\sqrt{2}}\cdot\begin{pmatrix} 1 \\ 1 \end{pmatrix} \quad X_d := \frac{1}{\sqrt{2}}\cdot\begin{pmatrix} 1 \\ -1 \end{pmatrix} \quad Y_u := \frac{1}{\sqrt{2}}\cdot\begin{pmatrix} 1 \\ i \end{pmatrix} \quad Y_d := \frac{1}{\sqrt{2}}\cdot\begin{pmatrix} 1 \\ -i \end{pmatrix}$$



This figure was taken from http://demonstrations.wolfram.com/QubitsOnThePoincareBlochSphere/ a contribution by Rudolf Muradian.

The Bloch sphere is prepared in Cartesian coordinates using Mathcad graphics.

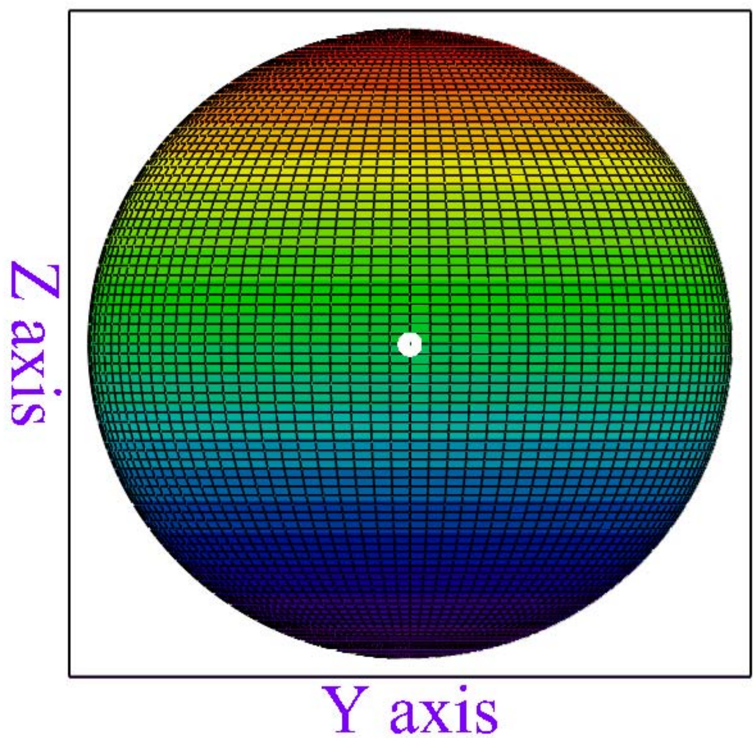
$$\text{numpts} := 100 \qquad i := 0..\text{numpts} \qquad j := 0..\text{numpts} \qquad \theta_i := \frac{\pi\cdot i}{\text{numpts}} \qquad \phi_j := \frac{2\cdot\pi\cdot j}{\text{numpts}}$$

$$X_{i,j} := \sin(\theta_i)\cdot\cos(\phi_j) \qquad Y_{i,j} := \sin(\theta_i)\cdot\sin(\phi_j) \qquad Z_{i,j} := \cos(\theta_i)$$

Next, the coordinates of a quantum qubit are calculated and displayed on the Bloch sphere as a white dot. As the polar and azmuthal angles are changed, you will need to rotate the figure to see where the white dot is on the surface of the Bloch sphere.

$$\theta 1 := \frac{\pi}{2} \qquad \phi 1 := 0 \qquad \Psi(\theta 1,\phi 1) := \cos\left(\frac{\theta 1}{2}\right)\cdot\begin{pmatrix} 1 \\ 0 \end{pmatrix} + \exp(i\cdot\phi 1)\cdot\sin\left(\frac{\theta 1}{2}\right)\cdot\begin{pmatrix} 0 \\ 1 \end{pmatrix} \qquad \Psi(\theta 1,\phi 1) = \begin{pmatrix} 0.707 \\ 0.707 \end{pmatrix}$$

$$XX_{i,j} := \sin(\theta 1)\cdot\cos(\phi 1) \qquad YY_{i,j} := \sin(\theta 1)\cdot\sin(\phi 1) \qquad ZZ_{i,j} := \cos(\theta 1)$$



(X, Y, Z), (XX, YY, ZZ)