

# Quantum Corrals: Electrons in a Ring

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"When electrons are confined to lengthscales approaching the de Broglie wavelength, their behavior is dominated by quantum mechanical effects. Here we report the construction and characterization of structures for confining electrons to this lengthscale. The walls of these "quantum corrals" are built from Fe adatoms which are individually positioned on the Cu (111) surface by means of a scanning tunneling microscope (STM). These adatom structures confine surface state electrons laterally because of the strong scattering that occurs between surface state electrons and the Fe adatoms. The surface state electrons are confined in the direction perpendicular to the surface because of intrinsic energetic barriers that exist in that direction."

This is the first paragraph of "Confinement of Electrons to Quantum Corrals on a Metal Surface," published by M. F. Crommie, C. P. Lutz, and D. M. Eigler in the October 8, 1993 issue of *Science Magazine*. They report the corralling of the surface electrons of Cu in a ring of radius  $135 a_0$  created by 48 Fe adatoms. The quantum mechanics for this form of electron confinement is well-known. Schrodinger's equation for a particle in a ring and its solution (in atomic units) are given below.

$$\frac{-1}{2 \cdot \mu} \cdot \frac{d^2}{dr^2} \Psi(r) - \frac{1}{2 \cdot r \cdot \mu} \cdot \frac{d}{dr} \Psi(r) + \left( \frac{L^2}{2 \cdot \mu \cdot r^2} \right) \cdot \Psi(r) = E \cdot \Psi(r)$$

$$E_{n,L} = \frac{(Z_{n,L})^2}{2 \cdot \mu \cdot R^2} \quad \Psi_{n,L}(r) = J_L(Z_{n,L}, R) \quad \text{unnormalized}$$

$J_L$  is the  $L^{\text{th}}$  order Bessel function,  $L$  is the angular momentum quantum number,  $n$  is the principle quantum number,  $Z_{n,L}$  is the  $n^{\text{th}}$  root of  $J_L$ ,  $\mu$  is the effective mass of the electron, and  $R$  is the corral (ring) radius. Dirac notation is used to describe the electronic states,  $|n,L\rangle$ . The roots of the Bessel function are given below in terms of the  $n$  and  $L$  quantum numbers.

		L quantum number								
		0	1	2	3	4	5	6	7	
$Z :=$	2.405	3.832	5.316	6.380	7.588	8.771	9.936	11.086		
	5.520	7.016	8.417	9.761	11.065	12.339	13.589	14.821		
	8.654	10.173	11.620	13.015	14.373	15.700	17.004	18.288		
	11.792	13.324	14.796	16.223	17.616	18.980	20.321	21.642		
	14.931	16.471	17.960	19.409	20.827	22.218	23.586	24.935		

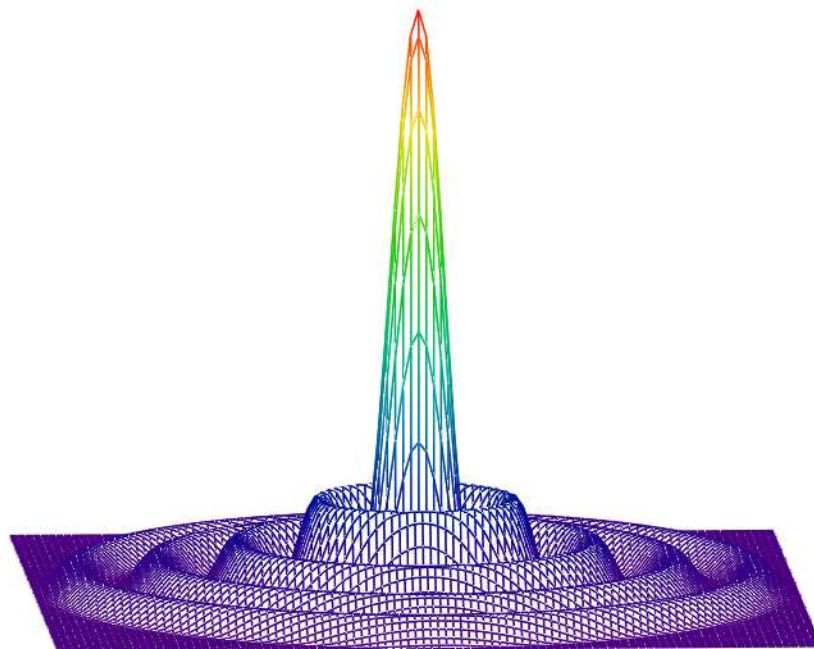
n quantum number

On the basis of Fermi energy considerations, Crombie, et al. identify the  $|5,0\rangle$ ,  $|4,2\rangle$  and  $|2,7\rangle$  as the most likely states contributing to the behavior of the surface electrons of Cu. A graphical comparison of the calculated surface electron density contributed by  $|5,0\rangle$  with the experimental data suggests that it is the dominant state in determining the surface electron density.

The theoretical results are displayed by plotting the wave function in cartesian coordinates.

$R := 135$        $N := 100$        $i := 0..N$        $j := 0..N$        $L := 0$        $n := 5$

$$x_i := -R + \frac{2 \cdot i}{N} \cdot R \quad y_j := -R + \frac{2 \cdot j}{N} \cdot R \quad \Psi(x, y) := \text{if} \left( \sqrt{x^2 + y^2} \leq R, \text{Jn} \left( L, Z_{n, L} \cdot \frac{\sqrt{x^2 + y^2}}{R} \right), 0 \right) \quad N_{i, j} := \Psi(x_i, y_j)^2$$



N

The experimental surface electron density reported by Crombie, et al. is shown below. The agreement between theory and experiment is very good.

