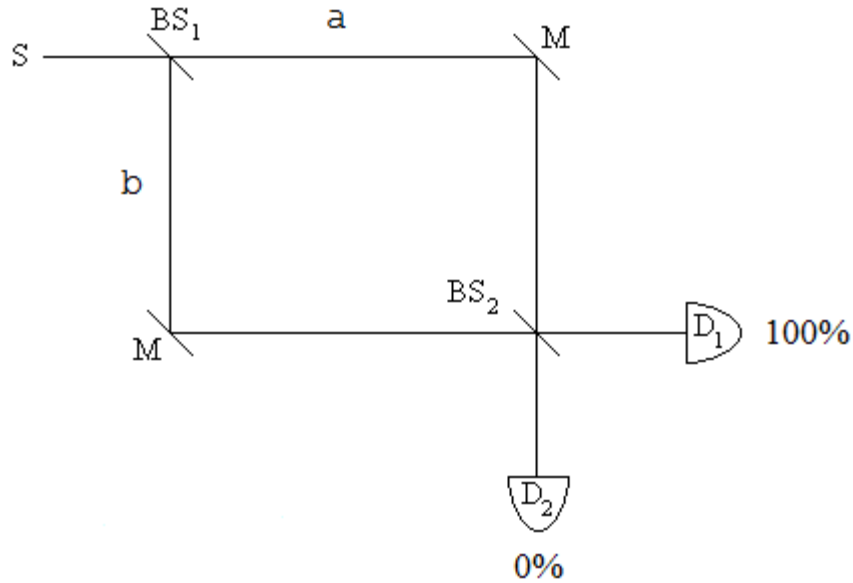


## Single-photon Interference in a Mach-Zehnder Interferometer



Photon wave function after first beam splitter:

$$|S\rangle \xrightarrow{BS1} \frac{1}{\sqrt{2}} [ |1,0\rangle_{a,b} + i |0,1\rangle_{a,b} ] = \frac{1}{\sqrt{2}} \left[ \begin{pmatrix} 0 \\ 1 \end{pmatrix} \otimes \begin{pmatrix} 1 \\ 0 \end{pmatrix} + i \begin{pmatrix} 1 \\ 0 \end{pmatrix} \otimes \begin{pmatrix} 0 \\ 1 \end{pmatrix} \right] = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ i \\ 1 \\ 0 \end{pmatrix} \quad \Psi := \frac{1}{\sqrt{2}} \cdot \begin{pmatrix} 0 \\ i \\ 1 \\ 0 \end{pmatrix}$$

### Operators

Creation	Annihilation	Number	Identity
$C := \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}$	$A := \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}$	$N := \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}$	$I := \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

Number operator for photon detection at D1: 
$$N_{D1} = \frac{-i\hat{a}^\dagger + \hat{b}^\dagger}{\sqrt{2}} \cdot \frac{i\hat{a} + \hat{b}}{\sqrt{2}}$$

$$N_{D1} := \frac{1}{2} \cdot (-i \text{kroncker}(C, I) + \text{kroncker}(I, C)) \cdot (i \cdot \text{kroncker}(A, I) + \text{kroncker}(I, A))$$

Number operator for photon detection at D2: 
$$N_{D2} = \frac{\hat{a}^\dagger - i\hat{b}^\dagger}{\sqrt{2}} \cdot \frac{\hat{a} + i\hat{b}}{\sqrt{2}}$$

$$N_{D2} := \frac{1}{2} \cdot (\text{kroncker}(C, I) - i \cdot \text{kroncker}(I, C)) \cdot (\text{kroncker}(A, I) + i \cdot \text{kroncker}(I, A))$$

Expectation value for photon detection at D1: 
$$\overline{(\Psi^T)} \cdot N_{D1} \cdot \Psi = 1$$

Expectation value for photon detection at D2: 
$$\overline{(\Psi^T)} \cdot N_{D2} \cdot \Psi = 0$$