Two Analyses of the Michelson Interferometer

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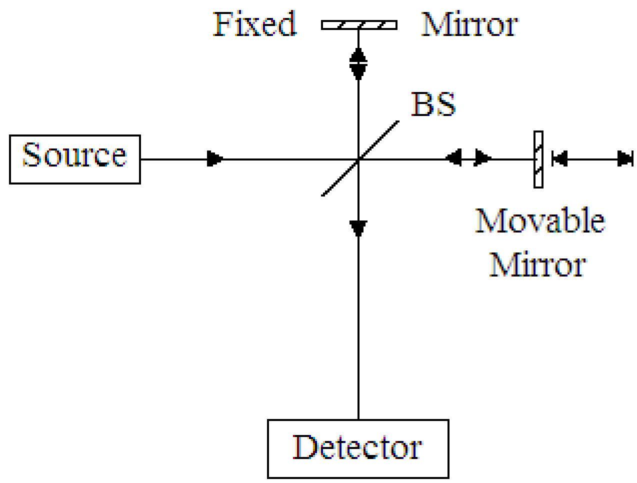

Path difference between interferometer arms is δ. Phase accumulated due to path difference: $\exp\left(i\cdot\frac{2\cdot\pi\cdot\delta}{\lambda}\right)$

S stands for source, **D** for detector, **T** for transmitted and **R** for reflected. The evolution of the photon wave function at various stages is given below.

$$S = \frac{1}{\sqrt{2}}\cdot(T + iR) \qquad T = \frac{\exp\left(i\cdot\frac{2\cdot\pi\cdot\delta}{\lambda}\right)}{\sqrt{2}}\cdot(i\cdot D + S) \qquad R = \frac{1}{\sqrt{2}}\cdot(D + i\cdot S)$$

$$S = \frac{1}{\sqrt{2}}\cdot(T + iR) \left| \begin{array}{l} \text{substitute}, T = \dfrac{\exp\left(i\cdot\frac{2\cdot\pi\cdot\delta}{\lambda}\right)}{\sqrt{2}}\cdot(i\cdot D + S) \\ \text{substitute}, R = \dfrac{1}{\sqrt{2}}\cdot(D + i\cdot S) \end{array} \right. \rightarrow S = -\frac{S}{2} + \frac{S\cdot e^{\frac{2i\cdot\pi\cdot\delta}{\lambda}}}{2} + \frac{D\cdot e^{\frac{2i\cdot\pi\cdot\delta}{\lambda}}\cdot i}{2} + \frac{D\cdot i}{2}$$

The probability the photon will arrive at the detector is the square of the absolute magnitude of the coefficient of D.

$$\frac{-1}{2}\cdot i\cdot\left(e^{-2\cdot i\cdot\pi\cdot\frac{\delta}{\lambda}} + 1\right)\cdot\frac{1}{2}\cdot i\cdot\left(e^{2\cdot i\cdot\pi\cdot\frac{\delta}{\lambda}} + 1\right) \text{ simplify } \rightarrow \cos\left(\frac{\pi\cdot\delta}{\lambda}\right)^2$$

The probability the photon will be returned to the source is the square of the absolute magnitude of the coefficient of S.

$$\frac{1}{2}\cdot\left(e^{\frac{-2i\cdot\pi\cdot\delta}{\lambda}} - 1\right)\cdot\frac{1}{2}\cdot\left(e^{\frac{2i\cdot\pi\cdot\delta}{\lambda}} - 1\right) \text{ simplify } \rightarrow \sin\left(\frac{\pi\cdot\delta}{\lambda}\right)^2$$

The same results are now illustrated using a matrix mechanics approach. Horizontal and vertical motion of the photon are represented by vectors. The source emits a horizontal photon, the detector receives a vertical photon. The beam splitter and the phase shift due to path length difference are represented by matrices. A matrix representation for the mirrors is unnecessary because the simply return the photon to the beam splitter.

Horizontal motion: $\begin{pmatrix} 1 \\ 0 \end{pmatrix}$ Vertical motion: $\begin{pmatrix} 0 \\ 1 \end{pmatrix}$

Beam splitter: $BS := \begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{i}{\sqrt{2}} \\ \frac{i}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix}$ Phase shift: $A(\delta) := \begin{pmatrix} e^{2 \cdot i \cdot \pi \cdot \frac{\delta}{\lambda}} & 0 \\ 0 & 1 \end{pmatrix}$

Calculate the probability amplitude and probability that the photon will arrive at the detector.

$$(0 \;\; 1) \cdot BS \cdot \begin{pmatrix} e^{2 \cdot i \cdot \pi \cdot \frac{\delta}{\lambda}} & 0 \\ 0 & 1 \end{pmatrix} \cdot BS \cdot \begin{pmatrix} 1 \\ 0 \end{pmatrix} \rightarrow \frac{e^{\frac{2i \cdot \pi \cdot \delta}{\lambda}} \cdot i}{2} + \frac{1}{2} \cdot i$$

$$\left(\frac{e^{\frac{-2i \cdot \pi \cdot \delta}{\lambda}} \cdot -i}{2} + \frac{1}{2} \cdot -i \right) \cdot \left(\frac{e^{\frac{2i \cdot \pi \cdot \delta}{\lambda}} \cdot i}{2} + \frac{1}{2} \cdot i \right) \text{ simplify } \rightarrow \cos\left(\frac{\pi \cdot \delta}{\lambda}\right)^2$$

Calculate the probability amplitude and probability that the photon will be returned to the source.

$$(1 \;\; 0) \cdot BS \cdot \begin{pmatrix} e^{2 \cdot i \cdot \pi \cdot \frac{\delta}{\lambda}} & 0 \\ 0 & 1 \end{pmatrix} \cdot BS \cdot \begin{pmatrix} 1 \\ 0 \end{pmatrix} \rightarrow \frac{e^{\frac{2i \cdot \pi \cdot \delta}{\lambda}}}{2} - \frac{1}{2}$$

$$\left(\frac{e^{\frac{-2i \cdot \pi \cdot \delta}{\lambda}}}{2} - \frac{1}{2} \right) \cdot \left(\frac{e^{\frac{2i \cdot \pi \cdot \delta}{\lambda}}}{2} - \frac{1}{2} \right) \text{ simplify } \rightarrow \sin\left(\frac{\pi \cdot \delta}{\lambda}\right)^2$$

Plotting these results in units of λ yields: $\delta := 0, .01 .. 1$