Quantum Teleportation: A Brief Outline

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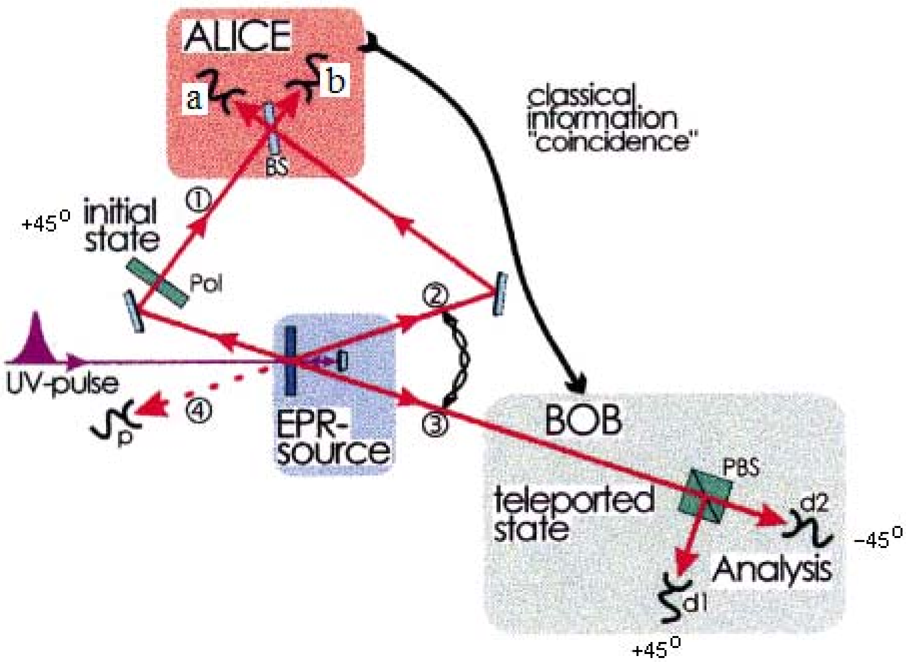
Alice wishes to teleport a photon in the polarization state, $\alpha \cdot h_1 + \beta \cdot v_1$, to Bob, where α and β are complex coefficients such that the sum of the square of their absolute magnitudes is unity. The letters ***h*** and ***v*** refer to the vertical and horizontal polarization states. In preparation for the teleportation event, Alice and Bob first prepare an entangled state involving photons 2 and 3, $\frac{h_2 \cdot v_3 - v_2 \cdot h_3}{\sqrt{2}}$, as shown in the figure below [*Nature* **390**, 576 (1997)].



Alice has photon 2 and Bob photon 3, but because they are in an entangled state the photons do not have well-defined individual polarization states.

Alice arranges for photons 1 and 2 to arrive at opposite sides of a beamsplitter at the same time. This gives rise to the following state,

$$\left(\alpha \cdot h_1 + \beta \cdot v_1\right) \cdot \frac{h_2 \cdot v_3 - v_2 \cdot h_3}{\sqrt{2}} \quad (1)$$

which upon expansion yields,

$$\frac{1}{2} \cdot \sqrt{2} \cdot \alpha \cdot h_1 \cdot h_2 \cdot v_3 - \frac{1}{2} \cdot \sqrt{2} \cdot \alpha \cdot h_1 \cdot v_2 \cdot h_3 + \frac{1}{2} \cdot \sqrt{2} \cdot \beta \cdot v_1 \cdot h_2 \cdot v_3 - \frac{1}{2} \cdot \sqrt{2} \cdot \beta \cdot v_1 \cdot v_2 \cdot h_3 \quad (2)$$

Alice now makes a Bell-state measurement (*vide infra*) at the detectors (***a*** and ***b***) to the left and right of her beamsplitter. Bell states are the four maximally entangled h-v polarization states of photons 1 and 2. They are as follows:

$$\Phi_p = \frac{h_1 \cdot h_2 + v_1 \cdot v_2}{\sqrt{2}} \qquad \Phi_m = \frac{h_1 \cdot h_2 - v_1 \cdot v_2}{\sqrt{2}} \qquad \Psi_p = \frac{h_1 \cdot v_2 + v_1 \cdot h_2}{\sqrt{2}} \qquad \Psi_m = \frac{h_1 \cdot v_2 - v_1 \cdot h_2}{\sqrt{2}} \qquad (3)$$

The products of the polarization states of photons 1 and 2 in equation (3) can be expressed as linear superpositions of the Bell states.

$$h_1 \cdot h_2 = \frac{\sqrt{2}}{2} \cdot (\Phi_p + \Phi_m) \qquad h_1 \cdot v_2 = \frac{\sqrt{2}}{2} \cdot (\Psi_p + \Psi_m) \qquad v_1 \cdot h_2 = \frac{\sqrt{2}}{2} \cdot (\Psi_p - \Psi_m) \qquad v_1 \cdot v_2 = \frac{\sqrt{2}}{2} \cdot (\Phi_p - \Phi_m) \qquad (4)$$

We can let Mathcad do the heavy lifting by having it expand (1), substitute equations (4) and collect on the Bell states.

$$(\alpha \cdot h_1 + \beta \cdot v_1) \cdot \frac{h_2 \cdot v_3 - v_2 \cdot h_3}{\sqrt{2}} \left| \begin{array}{l} \text{expand} \\ \text{substitute}, h_1 \cdot h_2 = \frac{\sqrt{2}}{2} \cdot (\Phi_p + \Phi_m) \\ \text{substitute}, h_1 \cdot v_2 = \frac{\sqrt{2}}{2} \cdot (\Psi_p + \Psi_m) \\ \text{substitute}, v_1 \cdot h_2 = \frac{\sqrt{2}}{2} \cdot (\Psi_p - \Psi_m) \\ \text{substitute}, v_1 \cdot v_2 = \frac{\sqrt{2}}{2} \cdot (\Phi_p - \Phi_m) \\ \text{collect}, \Phi_m, \Phi_p, \Psi_m, \Psi_p \end{array} \right. \rightarrow \left(\frac{\beta \cdot h_3}{2} + \frac{\alpha \cdot v_3}{2}\right) \cdot \Phi_m + \left(\frac{\alpha \cdot v_3}{2} - \frac{\beta \cdot h_3}{2}\right) \cdot \Phi_p + \left(-\frac{\alpha \cdot h_3}{2} - \frac{\beta \cdot v_3}{2}\right) \cdot \Psi_m + \left(\frac{\beta \cdot v_3}{2} - \frac{\alpha \cdot h_3}{2}\right) \cdot \Psi_p$$

Thus Alice's Bell-state measurement has four equally likely ($0.5^2 = 0.25$) outcomes. We will restrict our attention to the third term which says that if Alice measures Ψ_m, then Bob receives photon 1's polarization state without further action on Bob's part.

$$-(\alpha \cdot h_3 + \beta \cdot v_3) \cdot \Psi_m$$

This will occur, of course, 25% of the time. The other three possible measurement outcomes are more complicated to analyze and will not be discussed further here.

So we assume that Alice measures Ψ_m and communicates this to Bob by classical means so that he knows that his photon (#3) now has the polarization state of the original photon 1. But, how does Alice know that the results she observes at detectors ***a*** and ***b*** mean that photons 1 and 2 are in Bell state Ψ_m?

First the short, qualitative answer. Of the four Bell states, Ψ_m is the only one that is antisymmetric with respect to the interchange of the labels of the photons. Thus, in spite of the fact that photons individually are bosons, this entangled state is fermionic - collectively the photons are behaving as fermions. This means that they can't be in the same (measurement) state at the same time. If photon 1 is detected at ***a***, then photon 2 will be detected at ***b***. Therefore, if Alice observes ***a-b*** coincindences it means that photons 1 and 2 are in the Ψ_m Bell state and photon 1's polarization state has been teleported to Bob's photon (#3).

Bob confirms that he has received the polaization state of photon 1 using a polarizing beamsplitter as shown in the figure above. Suppose Alice encodes photon 1 with a 45^o polarization, then Bob sets his polarizing beamsplitter to detect $+45^o/-45^o$ polarized photons. A three-fold coincidence between detectors ***a***, ***b*** and Bob's $+45^o$ detector confirms teleportation. This was procedure employed in the original experiment published in Nature on December 11, 1997.