Quantum Computation with the Quirk Simulator

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The purpose of this tutorial is to build some basic quantum computer circuits using the Quirk simulator. As will be shown in the applications that follow, a quantum computer uses superpositions, entanglement and interference effects to carry out calculations more efficiently than a classical digital computer.

Quirk is very easy to use but I disagree with the way it presents output states in binary notation. See Am. J. Phys. **71**, 23-30 (2003) by David Mermin for a thorough presentation of the proper use of binary notation in quantum mechanics. In conventional notation |110 > = 6 as shown below, but in Quirk it represents 3.



In summary Quirk uses a reverse order of the binary digits for output states which is not consistent with common practice. For a three wire circuit with bottom, middle and top wires, Quirk writes output as |BMT > while traditional practice is to write it as |TMB >.

We begin with the quantum full adder circuit shown here. To access the Quirk quantum circuit builder double click on the hyperlink below the circuit.





Next we look at an example of parallel quantum computation. The following circuit shows that f(0) = f(3) = 1 and f(1) = f(2) = 0. The link immediately below the circuit shows how I simulate this circuit with Mathcad. The first Quirk link calculates f(3) = 1. The second Quirk link calculates f(x) using a superposition of all four values of x, illustrating quantum parallel computation.



<http://www.users.csbsju.edu/~frioux/q-intro/QuantumParallelComputation.pdf>





The following link introduces the Deutsch-Jozsa algorithm which provides an efficient method for determining whether f(x) is a balanced or constant function in one operation of the circuit.

<http://www.users.csbsju.edu/~frioux/q-intro/DeutschJozsaAlgorithmBrief.pdf>





Quantum teleportation is a uniquely quantum method for transferring a quantum state from one location to another. It does, however, require a classical communication channel.

<http://www.users.csbsju.edu/~frioux/q-intro/TeleportationUsingQuantumGatesD.pdf>



The implementation of this circuit in Quirk is:





The following somewhat more complicated circuit teleports without measurement required on the top wires. Remember, Quirk writes the output as |BMT >.





Here’s how to teleport two qubits:

<http://www.users.csbsju.edu/~frioux/q-intro/TeleportTwoQubits.pdf>



If you measure 0, 4, 8 or 12 no further action is required; the teleported qubits are on the bottom two wires. If you measure 3, 7, 11 or 15 the NOT operation is applied to wires 5 and 6. The other measurement outcomes require flipping either qubit 5 or 6, but will not be discussed because of the confusion created by Quirk’s non-standard output display.



Teleporation of entanglement is also possible. The following Quirk circuit teleports the entangled Bell state, [|00> + |11>]/√2, from the top two wires on the left to the bottom two wires on the right. The output is intelligible if you recall that Quirk reports output amplitudes as |BMT> which is the reverse of convention. The output state for this teleportation circuit is, (|00> + |11>)(|0> + |1>)(|0> + |1>)|0>/√8.





Entanglement swapping is another form of teleportation.

<http://www.users.csbsju.edu/~frioux/q-intro/EntanglementSwapBrief.pdf>

Photons 1 and 2 are entangled in the Φp Bell state and photons 3 and 4 are entangled in the ψm Bell state. The following Quirk circuit projects photons 2 and 3 onto Φm, thereby entangling 1 and 4 in the ψp Bell state.



In the following circuit photons 2 and 3 are projected onto all four Bell states entangling photons 1 and 4 in a superposition of the four Bell states.



The following link and circuit illustrate quantum superdense coding.

<http://www.users.csbsju.edu/~frioux/q-intro/SuperdenseCoding1.pdf>

Quantum computers are delicate and interactions with the environment can cause errors which must be corrected. Here’s one example of quantum error correction.

<http://www.users.csbsju.edu/~frioux/q-intro/CorrectQuantumError.pdf>

In the following circuit, the error is the NOT gate on the top wire. It causes a flip which is corrected by the remainder of the circuit.



In this circuit a phase error on the top wire (Z) is corrected.



Simon’s algorithm and its Quirk circuit are considered next.

<http://www.users.csbsju.edu/~frioux/q-intro/SimonsAlgorithm.pdf>



The next two circuits deal with Grover’s search algorithm. The first is a version of four-card Monte in which an item is hidden on the diagonal of a 4x4 matrix.

<http://www.users.csbsju.edu/~frioux/q-intro/FourCardMonte.pdf>



The next circuit finds two items that are hidden on the diagonal at positions 3 and 5 of an 8x8 matrix.

<http://www.users.csbsju.edu/~frioux/q-intro/GroverMonroe.pdf>

The initial density matrix shows the location of the hidden items and the output density matrix shows that the algorithm has correctly located the hidden items.



If implemented on a large quantum computer, Shor’s quantum algorithm for finding the prime factors of large integers will have enormous consequences for digital privacy and security. The following links show the quantum mechanics behind Shor’s algorithm; the first in mathematical detail and the second in a mainly algebraic format.

<http://www.users.csbsju.edu/~frioux/q-intro/ShorAlgorithm15.pdf>

<http://www.users.csbsju.edu/~frioux/q-intro/ShorAlgorithmSummary.pdf>

Here’s how the prime factors of 15 are found using Quirk.



The two output displays immediately after the QFT on the x-register show |0000>, |0100>, |1000> and |1100> (0, 4, 8, 12) revealing a period of 4. Using the Euclidian algorithm with B = 8 yields the prime factors 3 and 5.

The Greenberger-Horne-Zeilinger (GHZ) thought (gedanken) experiment, formulated in 1990, succinctly reveals the conflict between the local realistic and quantum views of reality. The following link summarizes the GHZ thought experiment and its simulation using quantum circuits.

<http://www.users.csbsju.edu/~frioux/q-intro/GHZ-Brief-Simulation.pdf>

The following Quirk circuits show that the input state is an eigenfunction of the xyy, yxy, yyx and xxx operators. For the first three the output state is identical to the input state, indicating an eigenvalue of +1. For xxx the output state is the negative of the input state indicating an eigenvalue of -1.









The following Quirk circuits show that the four-photon input state is an eigenfunction of the xxxx, xyxy, xxyy and xyyx operators. For the first three the output state is identical to the input state, indicating an eigenvalue of +1. For xyyx the output state is minus the input state indicating an eigenvalue of -1.

<http://www.users.csbsju.edu/~frioux/q-intro/GHZ4-Brief.pdf>









Of course it was John Bell in the 1960s who was the first to show that the conflict between quantum theory and local realism could be adjudicated in the laboratory.

<http://www.users.csbsju.edu/~frioux/q-intro/QuantumSimulationOfBellCorrelations.pdf>

A Quirk quantum circuit can be used to show that Bell’s inequality |E(a,b)-E(a,c)|-E(b,c) <=1 is violated. I recommend the following angles: a = 0 deg; b = 60 deg; c = 120 deg. Using these angles |E(a,b)-E(a,c)|-E(b,c) =1.5 in clear violation of Bell’s inequality. This circuit simulates E(a,b).



The following circuits simulate E(a,c) and E(b,c).





When the angle between the detectors is θ, the expectation value is E(θ) = -cos(θ). This is easily verified for the Quirk circuits given above.

All these GHZ and Bell thought experiments have dealt with spin ½ systems. But the experiments have actually been done with photons, most convincingly by GHZ and collaborators, and also by Alain Aspect and his research team in Paris. The following link briefly summarizes the results of Aspect’s experiments and presents a quantum computer simulation.

<http://www.users.csbsju.edu/~frioux/q-intro/QuantumSimulationPhotonCorrelations.pdf>



Richard Feynman explored the clash between classical reality and quantum mechanics with an analysis of positronium annihilation presented in Volume III (section 18-3) of *The Feynman Lectures on Physics*.

<http://www.users.csbsju.edu/~frioux/stability/PositroniumAnnihilationOutline.pdf>

All the measurement scenarios examined in this tutorial can be simulated with the following Quirk circuit. The required circuit gates can be found at the lower right in the section labeled Custom Gates.



In summary, the Quirk circuits presented in this tutorial show how a quantum computer calculates, teleports, searches, factors and simulates by using superpositions, entangled states and interference.