# Teleportation Using Quantum Gates

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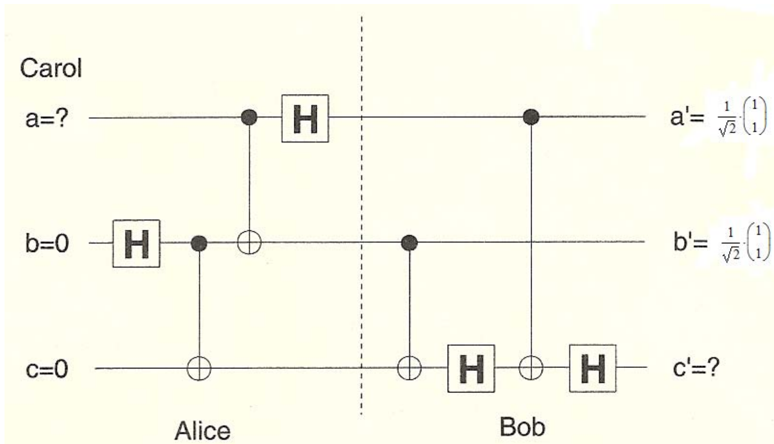
Implementation of the following 8-step circuit using quantum gates teleports Carol's cubit |? > to Bob. An alternative circuit is provided in the **Appendix A**.




In the matrix version of quantum mechanics, vectors represent states and matrices represent operators or, in this application, quantum gates. Quantum gates are required to be unitary matrices.

The necessary quantum bits or qubit states are:

Base states: A superposition of base states:

$$0 = \begin{pmatrix}1\\0\end{pmatrix} \qquad 1 = \begin{pmatrix}0\\1\end{pmatrix} \qquad \begin{pmatrix}\alpha\\\beta\end{pmatrix} = \alpha\cdot\begin{pmatrix}1\\0\end{pmatrix} + \beta\cdot\begin{pmatrix}0\\1\end{pmatrix} \quad \text{where} \quad (|\alpha|)^2 + (|\beta|)^2 = 1$$

The identity operator and the following quantum gates are required to calculate the result of the teleportation circuit displayed above. (See the **Appendix B** for the truth table for the Step-7 controlled-NOT gate.)

Identity — Hadamard gate — Controlled-NOT gate — Step-7 Controlled-NOT gate

$$I := \begin{pmatrix}1 & 0\\0 & 1\end{pmatrix} \qquad H := \frac{1}{\sqrt{2}}\cdot\begin{pmatrix}1 & 1\\1 & -1\end{pmatrix} \qquad CNOT := \begin{pmatrix}1&0&0&0\\0&1&0&0\\0&0&0&1\\0&0&1&0\end{pmatrix}$$

$$CnNOT := \begin{pmatrix}1&0&0&0&0&0&0&0\\0&1&0&0&0&0&0&0\\0&0&1&0&0&0&0&0\\0&0&0&1&0&0&0&0\\0&0&0&0&0&1&0&0\\0&0&0&0&1&0&0&0\\0&0&0&0&0&0&0&1\\0&0&0&0&0&0&1&0\end{pmatrix}$$

Using $\begin{pmatrix}\sqrt{\frac{2}{3}}\\ \sqrt{\frac{1}{3}}\end{pmatrix}$ as Carole's input qubit, the three-qubit initial state is,

$$\begin{pmatrix} \sqrt{2/3} \\ \sqrt{1/3} \end{pmatrix} \otimes \begin{pmatrix} 1 \\ 0 \end{pmatrix} \otimes \begin{pmatrix} 1 \\ 0 \end{pmatrix} = \begin{pmatrix} \sqrt{2/3} \\ 0 \\ 0 \\ 0 \\ \sqrt{1/3} \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

According to the teleporation circuit, the final state should be,

$$\frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ 1 \end{pmatrix} \otimes \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ 1 \end{pmatrix} \otimes \begin{pmatrix} \sqrt{2/3} \\ \sqrt{1/3} \end{pmatrix} = \begin{pmatrix} \sqrt{1/6} \\ \sqrt{1/12} \\ \sqrt{1/6} \\ \sqrt{1/12} \\ \sqrt{1/6} \\ \sqrt{1/12} \\ \sqrt{1/6} \\ \sqrt{1/12} \end{pmatrix}$$

where $\sqrt{\frac{1}{6}} = 0.408$  $\sqrt{\frac{1}{12}} = 0.289$

The following operations on the initial state indicated in the teleportation circuit yield the predicted final state, and the successful teleportation of Carol's cubit to Bob. In the Mathcad programming environment, *kronecker* is the command for matrix tensor multiplication.

Step1 := kronecker(I, kronecker(H, I))  Step2 := kronecker(I, CNOT)  Step3 := kronecker(CNOT, I)

Step4 := kronecker(H, kronecker(I, I))  Step5 := kronecker(I, CNOT)  Step6 := kronecker(I, kronecker(I, H))

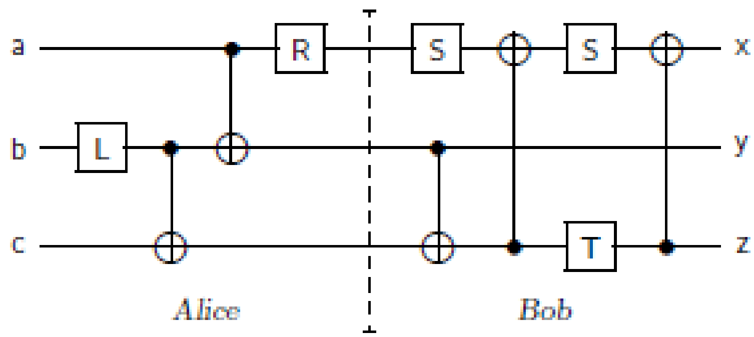
Step7 := CnNOT  Step8 := kronecker(I, kronecker(I, H))

$$\text{Step8}\cdot\text{Step7}\cdot\text{Step6}\cdot\text{Step5}\cdot\text{Step4}\cdot\text{Step3}\cdot\text{Step2}\cdot\text{Step1}\cdot\begin{pmatrix} \sqrt{\frac{2}{3}} \\ 0 \\ 0 \\ 0 \\ \sqrt{\frac{1}{3}} \\ 0 \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} 0.408 \\ 0.289 \\ 0.408 \\ 0.289 \\ 0.408 \\ 0.289 \\ 0.408 \\ 0.289 \end{pmatrix}$$

**Acknowledgments:** The quantum teleportation circuit was taken from page 226 of *The Quest for the Quantum Computer* by Julian Brown. The a' and b' product states in Brown's circuit are incorrect and have been corrected in this treatment. The error was detected by reading "Teleportation as a Quantum Computation" by Gilles Brassard (arXiv:quant-ph/9605035v1).

# Appendix A

Brassard provides the following teleportation circuit in "Teleportation as a Quantum Computation" (arXiv:quant-ph/9605035v1).




The initial state |a >|b >|c > and final state |x >|y >|z > are the same as those in the analysis presented above. The L, R, S and T gates are provided by Brassard.

$$L := \frac{1}{\sqrt{2}} \cdot \begin{pmatrix} 1 & -1 \\ 1 & 1 \end{pmatrix} \qquad R := \frac{1}{\sqrt{2}} \cdot \begin{pmatrix} 1 & 1 \\ -1 & 1 \end{pmatrix} \qquad S := \begin{pmatrix} i & 0 \\ 0 & 1 \end{pmatrix} \qquad T := \begin{pmatrix} -1 & 0 \\ 0 & -i \end{pmatrix}$$

The controlled-NOT gate required in steps 6 and 8 is,

$$ICnNOT := \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \end{pmatrix}$$

The truth table for the ICnNOT gate can be found in the **Appendix B**.

Step1 := kronecker(I, kronecker(L, I))    Step2 := kronecker(I, CNOT)    Step3 := kronecker(CNOT, I)

Step4 := kronecker(R, kronecker(I, I))    Step5 := kronecker(S, CNOT)    Step6 := ICnNOT

Step7 := kronecker(S, kronecker(I, T))    Step8 := ICnNOT

Feeding the initial state through the circuit yields the result predicted by Brassard. It was success with this calculation that led me to identify the error in Brown's results.

$$\mathrm{Step8}\cdot\mathrm{Step7}\cdot\mathrm{Step6}\cdot\mathrm{Step5}\cdot\mathrm{Step4}\cdot\mathrm{Step3}\cdot\mathrm{Step2}\cdot\mathrm{Step1}\cdot\begin{pmatrix}\sqrt{\frac{2}{3}}\\0\\0\\0\\\sqrt{\frac{1}{3}}\\0\\0\\0\end{pmatrix}=\begin{pmatrix}0.408\\0.289\\0.408\\0.289\\0.408\\0.289\\0.408\\0.289\end{pmatrix}$$

## Appendix B

In the controlled-NOT gate of step 7 **a** is the control and **c** is the target, while **b** is unaffected.

| a | b | c | ' | a' | b' | c' |
|---|---|---|---|---|---|---|
| 0 | 0 | 0 | ' | 0 | 0 | 0 |
| 0 | 0 | 1 | ' | 0 | 0 | 1 |
| 0 | 1 | 0 | ' | 0 | 1 | 0 |
| 0 | 1 | 1 | ' | 0 | 1 | 1 |
| 1 | 0 | 0 | ' | 1 | 0 | 1 |
| 1 | 0 | 1 | ' | 1 | 0 | 0 |
| 1 | 1 | 0 | ' | 1 | 1 | 1 |
| 1 | 1 | 1 | ' | 1 | 1 | 0 |

In the controlled-NOT gate of steps 6 and 8 of Brassard's teleportation circuit, **c** is the control, **a** is the target and **b** is unchanged.

| a | b | c | ' | a' | b' | c' |
|---|---|---|---|---|---|---|
| 0 | 0 | 0 | ' | 0 | 0 | 0 |
| 0 | 0 | 1 | ' | 1 | 0 | 1 |
| 0 | 1 | 0 | ' | 0 | 1 | 0 |
| 0 | 1 | 1 | ' | 1 | 1 | 1 |
| 1 | 0 | 0 | ' | 1 | 0 | 0 |
| 1 | 0 | 1 | ' | 0 | 0 | 1 |
| 1 | 1 | 0 | ' | 1 | 1 | 0 |
| 1 | 1 | 1 | ' | 0 | 1 | 1 |