Wave-particle Duality and the Uncertainty Principle

Nick Herbert, author of *Quantum Reality*, has proposed the name *quon* for "any entity that exhibits both wave and particle attributes in the quantum mechanical manner." Obvious examples of quons for chemists are the electron, proton, neutron and photon. The wave-particle duality of quons is captured succinctly by the deBroglie relation, which unites the wave property λ with the particle property *mv*.

$$\lambda = \frac{h}{mv} = \frac{h}{p}$$

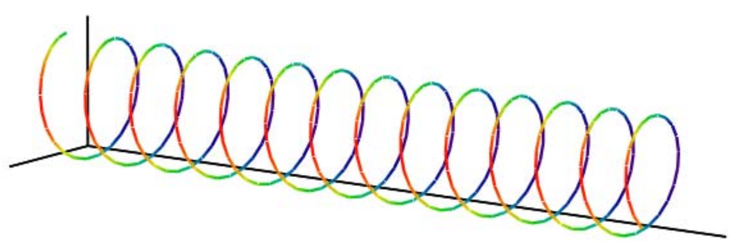
The most general momentum wave function for a quon in one-dimension is Euler's equation when it incorporates the deBroglie equation.

$$\langle x | p \rangle = \exp\left(i2\pi \frac{x}{\lambda}\right) = \exp\left(\frac{ipx}{\hbar}\right) = \cos\left(\frac{px}{\hbar}\right) + i\sin\left(\frac{px}{\hbar}\right)$$

The momentum wave function for a quon with a well-defined momentum (p = 7, for example) is shown below in atomic units (h = 2π). It clearly illustrates the uncertainty principle because the wave function is completely spatially delocalized.

Momentum: $p := 7$ $\quad j := 0 .. 300$ $\quad$ Axis of propagation: $x_j := j \cdot .04$

Real axis: $y_j := \mathrm{Re}\left(\exp\left(i \cdot p \cdot x_j\right)\right)$ $\quad$ Imaginary axis: $z_j := \mathrm{Im}\left(\exp\left(i \cdot p \cdot x_j\right)\right)$



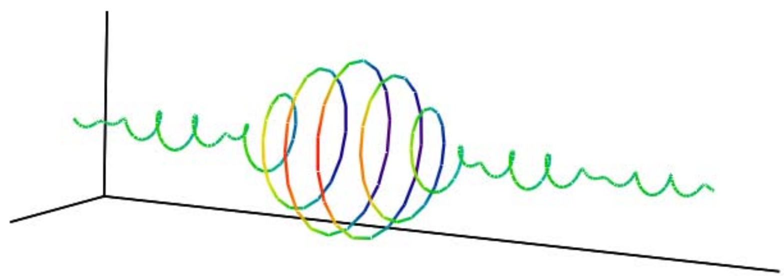
(x , y , z)

As might be expected on the basis of the uncertainty principle, the particle-like character of a quon is revealed only when there is uncertainty in momentum. This can be demonstrated by plotting Euler's equation for a superposition of momentum states as shown below. This superposition (integrating, or summing over a range of momentum values) clearly reveals the incipient particle-like characteristics of a quon.

Axis of propagation: $x_j := j \cdot .054 - 7$

Real axis: $y_j := \int_6^8 \mathrm{Re}\left(\exp\left(i \cdot p \cdot x_j\right)\right) dp$

Imaginary axis: $z_j := \int_6^8 \mathrm{Im}\left(\exp\left(i \cdot p \cdot x_j\right)\right) dp$



(x, y, z)