

The de Broglie-Bohr Model for the Hydrogen Atom

$$\lambda = \frac{h}{m \cdot v}$$

de Broglie's hypothesis that matter has wave-like properties.

$$n \cdot \lambda = 2 \cdot \pi \cdot r$$

The consequence of de Broglie's hypothesis; an integral number of wavelengths must fit within the circumference of the orbit. This introduces the quantum number which can have values 1,2,3,...

$$m \cdot v = \frac{n \cdot h}{2 \cdot \pi \cdot r}$$

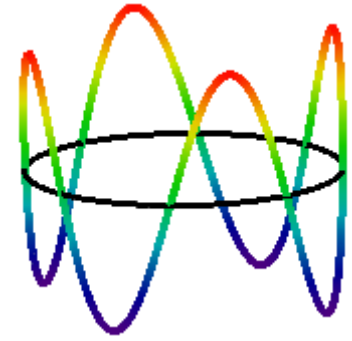
Substitution of the first equation into the second equation reveals that linear momentum is quantized.

$$T = \frac{1}{2} \cdot m \cdot v^2 = \frac{n^2 \cdot h^2}{8 \cdot \pi^2 \cdot m_e \cdot r^2}$$

If momentum is quantized, so is kinetic energy.

$$E = T + V = \frac{n^2 \cdot h^2}{8 \cdot \pi^2 \cdot m_e \cdot r^2} - \frac{e^2}{4 \cdot \pi \cdot \epsilon_0 \cdot r}$$

Which means that total energy is quantized.



Below the ground state energy and orbit radius of the electron in the hydrogen atom is found by plotting the energy as a function of the orbital radius. The ground state is the minimum in the curve.

Fundamental constants: electron charge, electron mass, Planck's constant, vacuum permittivity.

$$e := 1.6021777 \cdot 10^{-19} \cdot \text{coul}$$

$$m_e := 9.10939 \cdot 10^{-31} \cdot \text{kg}$$

$$h := 6.62608 \cdot 10^{-34} \cdot \text{joule} \cdot \text{sec}$$

$$\epsilon_0 := 8.85419 \cdot 10^{-12} \cdot \frac{\text{coul}^2}{\text{joule} \cdot \text{m}}$$

Quantum number and conversion factor between meters and picometers and joules and attojoules.

$$n := 1$$

$$\text{pm} := 10^{-12} \cdot \text{m}$$

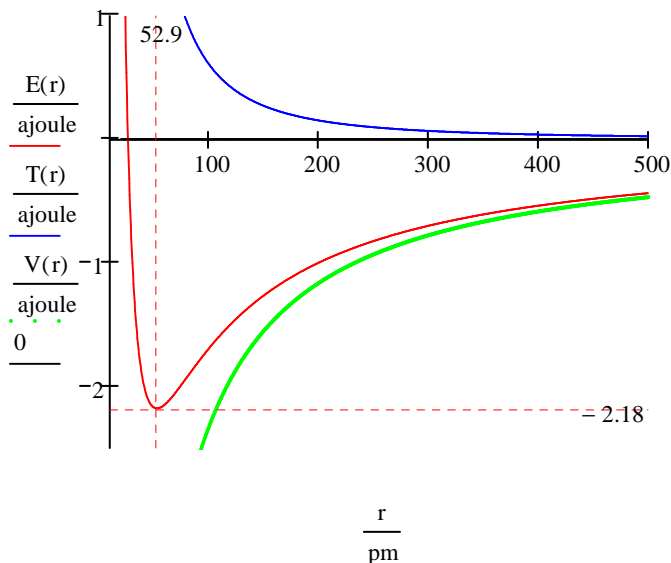
$$\text{ajoule} := 10^{-18} \cdot \text{joule}$$

$$r := 20 \cdot \text{pm}, 20.5 \cdot \text{pm} \dots 500 \cdot \text{pm}$$

$$T(r) := \frac{n^2 \cdot h^2}{8 \cdot \pi^2 \cdot m_e \cdot r^2}$$

$$V(r) := -\frac{e^2}{4 \cdot \pi \cdot \epsilon_0 \cdot r}$$

$$E(r) := T(r) + V(r)$$



This figure shows that atomic stability involves a balance between potential and kinetic energy. The electron is drawn toward the nucleus by the attractive potential energy interaction ($\sim -1/R$), but is prevented from spiraling into the nucleus by the extremely large kinetic energy ($\sim 1/R^2$) associated with small orbits.