

The deBroglie-Bohr Model for the Hydrogen Atom

$$\lambda = \frac{h}{m \cdot v}$$

de Broglie's hypothesis that matter has wave-like properties.

$$n \cdot \lambda = 2 \cdot \pi \cdot r$$

The consequence of de Broglie's hypothesis; an integral number of wavelengths must fit within the circumference of the orbit. This introduces the quantum number, n, which can have values 1,2,3,...

$$m \cdot v = \frac{n \cdot h}{2 \cdot \pi \cdot r}$$

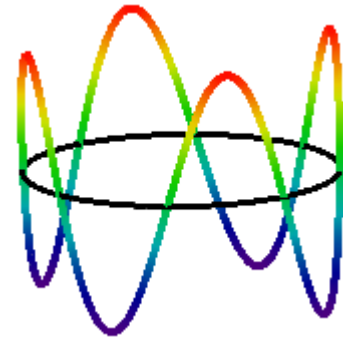
Substitution of the first equation into the second equation reveals that linear momentum is quantized.

$$T = \frac{1}{2} \cdot m \cdot v^2 = \frac{n^2 \cdot h^2}{8 \cdot \pi^2 \cdot m_e \cdot r^2}$$

If momentum is quantized, so is kinetic energy.

$$E = T + V = \frac{n^2 \cdot h^2}{8 \cdot \pi^2 \cdot m_e \cdot r^2} - \frac{Z \cdot q^2}{4 \cdot \pi \cdot \epsilon_0 \cdot r}$$

Which means that total energy is quantized.



$$\frac{d}{dr} \left(\frac{n^2 \cdot h^2}{8 \cdot \pi^2 \cdot m_e \cdot r^2} - \frac{Z \cdot q^2}{4 \cdot \pi \cdot \epsilon_0 \cdot r} \right) = 0 \text{ solve, } r \rightarrow n^2 \cdot h^2 \cdot \frac{\epsilon_0}{Z \cdot q^2 \cdot \pi \cdot m_e}$$

Minimization of the energy with respect to orbit radius yields the optimum value of r. This expression is substituted back in the energy expression below to find the optimum energy.

$$E = \frac{n^2 \cdot h^2}{8 \cdot \pi^2 \cdot m_e \cdot r^2} - \frac{Z \cdot q^2}{4 \cdot \pi \cdot \epsilon_0 \cdot r} \text{ substitute, } r = n^2 \cdot h^2 \cdot \frac{\epsilon_0}{Z \cdot q^2 \cdot \pi \cdot m_e} \rightarrow E = \frac{-1}{8 \cdot n^2 \cdot h^2} \cdot \frac{m_e}{\epsilon_0^2} \cdot Z^2 \cdot q^4$$

Fundamental constants: electron charge, electron mass, Planck's constant, vacuum permittivity.

$$q := 1.6021777 \cdot 10^{-19} \cdot \text{coul}$$

$$m_e := 9.10939 \cdot 10^{-31} \cdot \text{kg}$$

$$h := 6.62608 \cdot 10^{-34} \cdot \text{joule} \cdot \text{sec}$$

$$\epsilon_0 := 8.85419 \cdot 10^{-12} \cdot \frac{\text{coul}^2}{\text{joule} \cdot \text{m}}$$

Conversion factors between meters and picometers and joules and atto joules.

$$\text{pm} := 10^{-12} \cdot \text{m} \quad \text{aJ} := 10^{-18} \cdot \text{joule}$$

$$\text{eV} := 1.602177 \cdot 10^{-19} \cdot \text{joule}$$

Nuclear charge: $Z := 1$

Energy:

$$E(n) := \frac{-1}{8 \cdot n^2 \cdot h^2} \cdot \frac{m_e}{\epsilon_0^2} \cdot Z^2 \cdot q^4$$

Orbit radius:

$$r(n) := n^2 \cdot h^2 \cdot \frac{\epsilon_0}{Z \cdot q^2 \cdot \pi \cdot m_e}$$

Calculate first four energy levels and orbit radii.

$$n := 1..4 \quad \frac{E(n)}{\text{aJ}} = \begin{pmatrix} -2.180 \\ -0.545 \\ -0.242 \\ -0.136 \end{pmatrix} \quad \frac{r(n)}{\text{pm}} = \begin{pmatrix} 52.918 \\ 211.671 \\ 476.260 \\ 846.685 \end{pmatrix}$$