

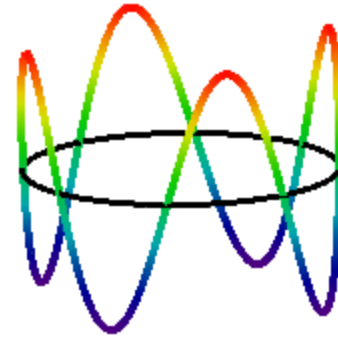
The deBroglie-Bohr Model for a Hydrogen Atom Held Together by a Gravitational Interaction

$$\lambda = \frac{h}{m \cdot v}$$

de Broglie's hypothesis that matter has wave-like properties.

$$n \cdot \lambda = 2 \cdot \pi \cdot r$$

The consequence of de Broglie's hypothesis; an integral number of wavelengths must fit within the circumference of the orbit. This introduces the quantum number, n, which can have values 1,2,3,...



$$m \cdot v = \frac{n \cdot h}{2 \cdot \pi \cdot r}$$

Substitution of the first equation into the second equation reveals that linear momentum is quantized.

$$T = \frac{1}{2} \cdot m \cdot v^2 = \frac{n^2 \cdot h^2}{8 \cdot \pi^2 \cdot m_e \cdot r^2}$$

If momentum is quantized, so is kinetic energy.

$$E = T + V = \frac{n^2 \cdot h^2}{8 \cdot \pi^2 \cdot m_e \cdot r^2} - \frac{G \cdot m_p \cdot m_e}{r}$$

Which means that total energy is quantized, where $-\frac{G \cdot m_p \cdot m_e}{r}$ is the gravitational potential energy interaction between a proton and an electron.

$$\frac{d}{dr} \left(\frac{n^2 \cdot h^2}{8 \cdot \pi^2 \cdot m_e \cdot r^2} - \frac{G \cdot m_p \cdot m_e}{r} \right) = 0 \text{ solve, } r \rightarrow \frac{h^2 \cdot n^2}{4 \cdot \pi^2 \cdot G \cdot m_e^2 \cdot m_p}$$

Minimization of the energy with respect to orbit radius yields the optimum values of r. This expression is substituted back in the energy expression below to find the allowed energies.

$$E = \frac{n^2 \cdot h^2}{8 \cdot \pi^2 \cdot m_e \cdot r^2} - \frac{G \cdot m_p \cdot m_e}{r} \text{ substitute, } r = \frac{h^2 \cdot n^2}{4 \cdot \pi^2 \cdot G \cdot m_e^2 \cdot m_p} \rightarrow E = -\frac{2 \cdot \pi^2 \cdot G^2 \cdot m_e^3 \cdot m_p^2}{h^2 \cdot n^2}$$

Fundamental constants:

$$m_p := 1.67262 \cdot 10^{-27} \cdot \text{kg}$$

$$m_e := 9.10939 \cdot 10^{-31} \cdot \text{kg}$$

$$h := 6.62608 \cdot 10^{-34} \cdot \text{joule} \cdot \text{sec} \quad G := 6.67259 \cdot 10^{-11} \cdot \frac{\text{m}^3}{\text{kg} \cdot \text{s}^2}$$

Energy:
$$E(n) := -\frac{2 \cdot \pi^2 \cdot G^2 \cdot m_e^3 \cdot m_p^2}{h^2 \cdot n^2}$$

Orbit radius:
$$r(n) := \frac{h^2 \cdot n^2}{4 \cdot \pi^2 \cdot G \cdot m_e^2 \cdot m_p}$$

Calculate first four energy levels and orbit radii.

$$n := 1..4 \quad \frac{E(n)}{\text{J}} = \begin{pmatrix} -4.233 \times 10^{-97} \\ -1.058 \times 10^{-97} \\ -4.704 \times 10^{-98} \\ -2.646 \times 10^{-98} \end{pmatrix} \quad \frac{r(n)}{\text{m}} = \begin{pmatrix} 1.201 \times 10^{29} \\ 4.803 \times 10^{29} \\ 1.081 \times 10^{30} \\ 1.921 \times 10^{30} \end{pmatrix}$$