Using the Mach-Zehnder Interferometer to
Illustrate the Impact of Which-way Information

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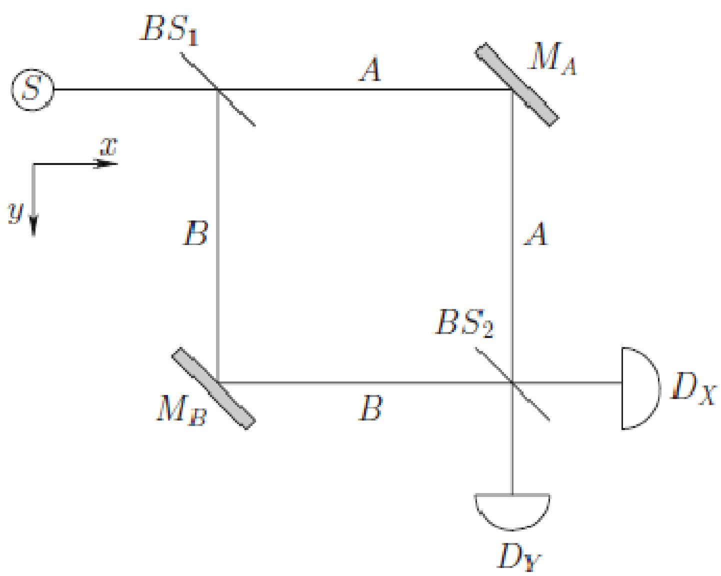
Richard Feynman raised Young's double-slit experiment to canonical status by presenting it as the paradigm for all quantum mechanical behavior. In *The Character of Physical Law* he wrote, "Any other situation in quantum mechanics, it turns out, can always be explained by saying, 'You remember the case of the experiment with the two holes? It's the same thing.'"

Following Feynman, teachers of quantum theory use the double-slit experiment to illustrate the superposition principle and its signature effect, quantum interference. A single particle (photon, electron, etc.) arrives at any point on the detection screen by two paths, whose probability amplitudes interfere yielding the characteristic diffraction pattern. This is called single-particle interference.

Single-particle interference in a Mach-Zehnder (MZ) interferometer is a close cousin of the traditional double-slit experiment. Using routine complex number algebra, it can be used to illustrate the same fundamentals as the two-slit experiment and also to introduce students to the field of quantum optics.

This tutorial draws heavily on a recent article in the American Journal of Physics and papers quoted therein [*Am. J. Phys.* 78(8), 792-795 (2010)]. An equal-arm MZ interferometer is shown below. In this configuration the photon is always detected at D_x. The analysis below provides an explanation why this happens.



A key convention in the analysis of MZ interferometers is that reflection at a beam splitter (BS) or mirror (M) is accompanied by a 90 degree phase shift (π/2, i). The behavior of a photon traveling in the x- or y-direction at the beam splitters and mirrors is as follows.

At the 50-50 beam splitters the following photon superpositions are formed:

$$|x\rangle \rightarrow \frac{1}{\sqrt{2}}\left[|x\rangle + i|y\rangle\right] \qquad |y\rangle \rightarrow \frac{1}{\sqrt{2}}\left[|y\rangle + i|x\rangle\right]$$

At the mirrors the behavior of the photon is as follows:

$$|x\rangle \rightarrow i|y\rangle \qquad |y\rangle \rightarrow i|x\rangle$$

Using the information provided above and complex number algebra, the history of a photon leaving the source (moving in the x-direction) is:

$$|S\rangle \xrightarrow{BS_1} \frac{1}{\sqrt{2}}\left[|x\rangle + i|y\rangle\right] \xrightarrow{Mirrors} \frac{1}{\sqrt{2}}\left[i|y\rangle - |x\rangle\right] \xrightarrow{BS_2} -|x\rangle = e^{i\pi}|x\rangle$$

Thus we see that, indeed, the photon always arrives at D_x in the equal-arm MZ interferometer shown above. The paths to D_x (TR+RT) are in phase and constructively interfere. The paths to D_y (TT+RR) are 180 degrees (i^2) out of phase and therefore destructively interfere. (T stands for transmitted and R stands for reflected.)

$$\text{Probability}\left(D_x\right) = \left|\langle x|S\rangle\right|^2 = 1 \qquad \text{Probability}\left(D_y\right) = \left|\langle y|S\rangle\right|^2 = 0$$

The detection of the photon exclusively at D_x is the equivalent of the appearance of the interference fringes in the double-slit experiment.

Another quantum mechanical point that Feynman made with the double-slit experiment is that if path information (which slit the photon went through) is available (even in principle) the interference fringes disappear. This is also the case with the MZ interferometer.

If after the first beam splitter the photon is observed in path A, we have the following history,

$$|S\rangle \xrightarrow{PathA} |x\rangle \xrightarrow{MirrorA} i|y\rangle \xrightarrow{BS_2} \frac{i}{\sqrt{2}}\left(|y\rangle + i|x\rangle\right)$$

which leads to equal probabilities of detecting the photon at D_x and D_y.

$$P(D_x) = \left|\langle x|S\rangle\right|^2 = \left|-\frac{1}{\sqrt{2}}\right|^2 = \frac{1}{2} \qquad P(D_y) = \left|\langle y|S\rangle\right|^2 = \left|\frac{i}{\sqrt{2}}\right|^2 = \frac{1}{2}$$

Alternatively, if after the first beam splitter the photon is observed in path B, we have the following history,

$$\left|S\right\rangle \xrightarrow{PathB} \left|y\right\rangle \xrightarrow{MirrorB} i\left|x\right\rangle \xrightarrow{BS_2} \frac{i}{\sqrt{2}}\left(\left|x\right\rangle + i\left|y\right\rangle\right)$$

which also leads to equal probabilities of detecting the photon at D_x and D_y.

$$P(D_x) = \left|\left\langle x\middle|S\right\rangle\right|^2 = \left|\frac{i}{\sqrt{2}}\right|^2 = \frac{1}{2} \qquad P(D_y) = \left|\left\langle y\middle|S\right\rangle\right|^2 = \left|-\frac{1}{\sqrt{2}}\right|^2 = \frac{1}{2}$$

In these cases, where path information is available the detection of the photon at both detectors in equal percentages is the equivalent of the disappearance of the interference fringes in the double-slit experiment when knowledge of which slit the particle went through is available.