# A Stern-Gerlach Quantum Eraser

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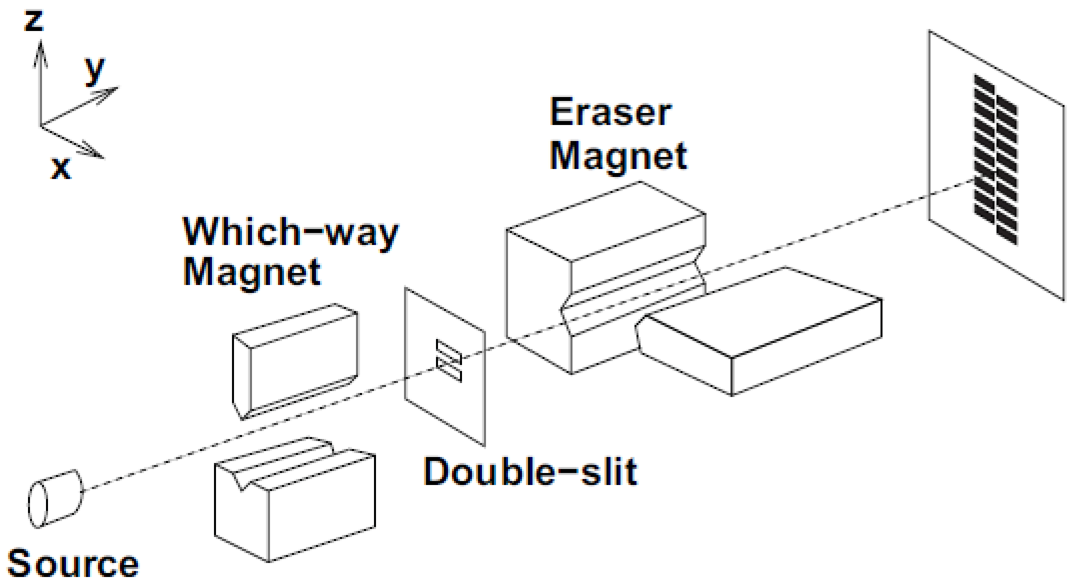
This tutorial provides a brief alternative mathematical analysis of the quantum eraser experiment shown below which is published at arXiv:quant-ph/0501010v2. Please see the two immediately preceding tutorials for another example of the quantum eraser and additional mathematical detail.




The first magnet attaches which-way information such that the beam leaving the double-slit screen is described by the following wave function,

$$|\Psi\rangle = \frac{1}{\sqrt{2}}\left[|\uparrow_z\rangle|z_1\rangle + |\downarrow_z\rangle|z_2\rangle\right]$$

where $z_1$ and $z_2$ represent the positions of the horizontal slits on the z-axis and the spin eigen states in the z-direction are given below.

$$\Psi_{zup} := \begin{pmatrix} 1 \\ 0 \end{pmatrix} \qquad \Psi_{zdown} := \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

Recognizing that a diffraction pattern is actually a momentum distribution function, we project Ψ onto momentum space as follows (in atomic units, h = 2π).

$$\langle p|\Psi\rangle = \frac{1}{\sqrt{2}}\left[|\uparrow_z\rangle\langle p|z_1\rangle + |\downarrow_z\rangle\langle p|z_2\rangle\right] = \frac{1}{\sqrt{2}}\left[|\uparrow_z\rangle\frac{\exp(-ipz_1)}{\sqrt{2\pi}} + |\downarrow_z\rangle\frac{\exp(-ipz_2)}{\sqrt{2\pi}}\right]$$

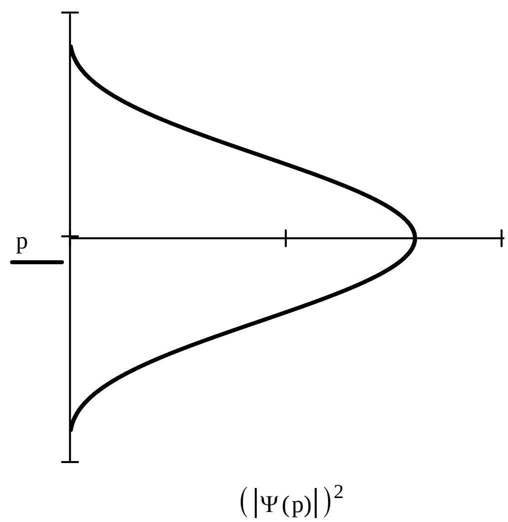
Here the exponential terms are the position eigenfunctions in momentum space for infinetesimally thin slits located at $z_1$ and $z_2$. For slits of finite width <p|Ψ> is written as shown below. Again see the previous tutorials in this series for further mathematical detail. The slit positions and slit width chosen are arbitrary.

Slit positions: $z_1 := 1$ $z_2 := 2$ Slit width: $\delta := .2$

$$\Psi(p) := \frac{1}{\sqrt{2}} \cdot \left( \Psi_{zup} \cdot \int_{z_1 - \frac{\delta}{2}}^{z_1 + \frac{\delta}{2}} \frac{1}{\sqrt{2 \cdot \pi}} \cdot \exp(-i \cdot p \cdot z) \cdot \frac{1}{\sqrt{\delta}}\, dz + \Psi_{zdown} \cdot \int_{z_2 - \frac{\delta}{2}}^{z_2 + \frac{\delta}{2}} \frac{1}{\sqrt{2 \cdot \pi}} \cdot \exp(-i \cdot p \cdot z) \cdot \frac{1}{\sqrt{\delta}}\, dz \right)$$

Because of the encoding of path information there are no interference fringes associated with this two-slit wave function. The encoded orthogonal z-direction eigen states destroy the interference cross terms as shown graphically below.

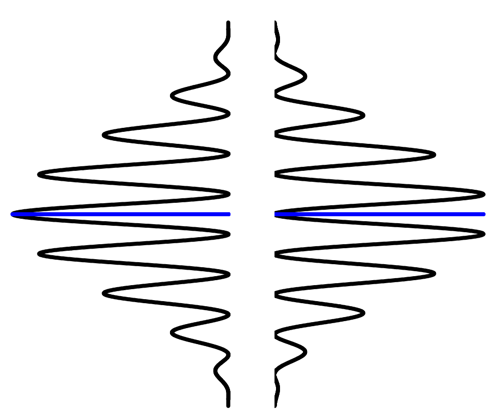
$$p := -30, -29.98 .. 30$$



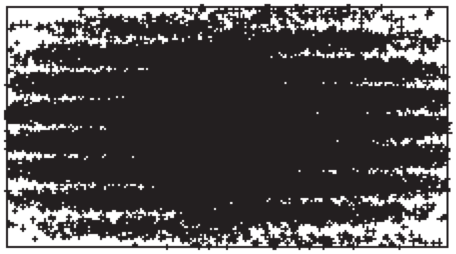

However, the second Stern-Gerlach magnet oriented in the x-direction erases the path information. This is shown by projecting the state after the first magnet and the slit screen, Ψ(p), onto the x-direction spin eigen states.

$$\Psi_{xup} := \frac{1}{\sqrt{2}} \cdot (1 \quad 1) \qquad \Psi_{xdown} := \frac{1}{\sqrt{2}} \cdot (1 \quad -1)$$

$$\Psi_{left}(p) := \Psi_{xup} \cdot \Psi(p) \qquad \Psi_{right}(p) := \Psi_{xdown} \cdot \Psi(p)$$



(b)



The two halves of this figure capture the basic features of Figure 2b in the reference cited and shown above. The horizontal blue line marks p = 0 on the z-axis. On the left is the interference pattern of the part of the beam emerging from the x-up magnet direction, with spin state $\Psi_{xup}$ which is an in-phase superposition of $\Psi_{zup}$ and $\Psi_{zdown}$ erasing which-way information. On the right is the interference pattern of the part of the beam emerging from the x-down magnet direction, with spin state $\Psi_{xdown}$ which is an out-of-phase superposition of $\Psi_{zup}$ and $\Psi_{zdown}$ also erasing which-way information. The relative vertical shift observed in the interference fringes is caused by the phase difference between the the two superposition states, $\Psi_{xup}$ and $\Psi_{xdown}$.