An Analysis of Three-Path Interference

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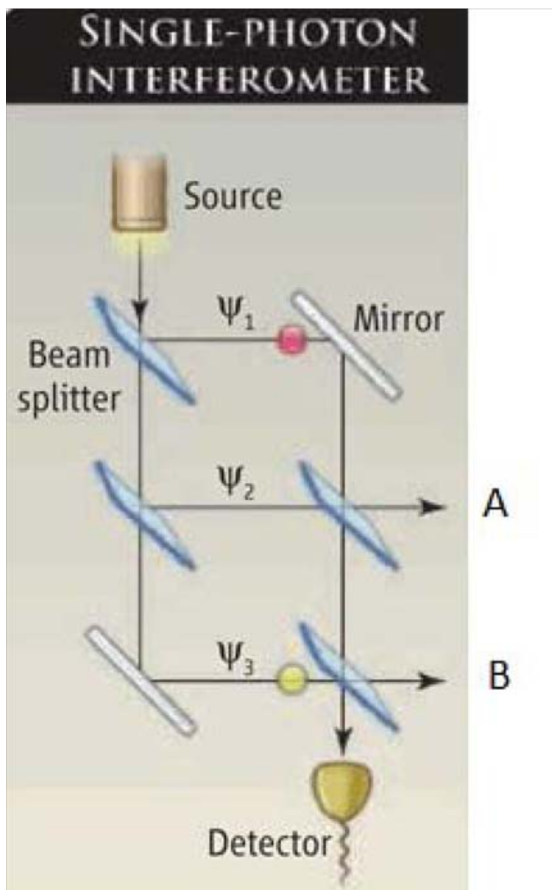
Quantum mechanics teaches that if there is more than one path to a particular destination interference effects are likely. In such cases the probability of arrival at a location is the square of the magnitude of the sum of the probability amplitudes for each path to that location. For example, in a triple-slit diffraction experiment the probability of arriving at **x** on the detection screen written in Dirac notation is,

$$P_{123} = \left|\langle x|S\rangle\right|^2 = \left|\langle x|3\rangle\langle 3|S\rangle + \langle x|2\rangle\langle 2|S\rangle + \langle x|1\rangle\langle 1|S\rangle\right|^2$$

The single-photon interferometer shown below [Franson, Science **329**, 396 (2010)] is a close cousin of the triple-slit experiment because it provides three paths to the detector. Initially we will ignore the other two output channels **A** and **B**.

Three Path Interference



Probability Amplitudes

In this analysis the probability amplitudes are calculate on the following conventions.

Assume 50-50 beam splitters and assign π/2 (i) phase shift to reflection (usual convention).

Transmission at a beam splitter: $T := \frac{1}{\sqrt{2}}$ Reflection at a beam splitter: $R := \frac{i}{\sqrt{2}}$

Reflection at a mirror: $M := i$

Calculate the probability that the photon will arrive at the **Detector** (3 paths):

$$\left(\left|R\cdot M\cdot T\cdot T + T\cdot R\cdot R\cdot T + T\cdot T\cdot M\cdot R\right|\right)^2 = 0.9161$$

To establish that probability is conserved, the probabilities for arrival at **A** and **B**.

Calculate the probability that the photon will arrive at **A** (2 paths):

$$\left(\left|T\cdot R\cdot T + R\cdot M\cdot R\right|\right)^2 = 0.0214$$

Calculate the probability that the photon will arrive at **B** (3 paths):

$$\left(\left|R\cdot M\cdot T\cdot R + T\cdot R\cdot R\cdot R + T\cdot T\cdot M\cdot T\right|\right)^2 = 0.0625$$

Demonstrate that probability is conserved: $0.9161 + .0214 + .0625 = 1$

While this traditional analysis postulates three-path interference for the arrival probability at the detector, Sinha et al. [Science 329, 418 (2010)] argue that true interference only occurs between pairs of paths. In this case between paths 1 & 2, 1 & 3, and 2 & 3. Franson summarized this view as follows: "Quantum interference between many different pathways is simply the sum of the effects from all pairs of pathways."

The probability expression for an event involving two equivalent paths is

$$P_{ij} = \left|\Psi_i + \Psi_j\right|^2 = \left|\Psi_i\right|^2 + \left|\Psi_j\right|^2 + \Psi_i^*\Psi_j + \Psi_j^*\Psi_i = P_i + P_j + I_{ij}$$

where I_{ij} is the *interference term* and P_i is defined as the probability when only the i^{th} path is open. It is my opinion that this latter designation is not strictly valid. However, accepting it for the time being leads to the following definition for two-path interference.

$$I_{ij} = P_{ij} - P_i - P_j$$

By similar arguments the probability expression for an event involving three equivalent paths is,

$$P_{123} = P_1 + P_2 + P_3 + I_{12} + I_{13} + I_{23}$$

Using this equation to define third-order interference yields,

$$I_{123} = P_{123} - \left(P_1 + P_2 + P_3 + I_{12} + I_{13} + I_{23}\right)$$

Using the definition for I_{ij} we obtain,

$$I_{123} = P_{123} + P_1 + P_2 + P_3 - P_{12} - P_{13} - P_{13}$$

From above $P_{123} = 0.9164$. The remaining terms in this equation are calculate below:

$P_1 \quad (|R \cdot M \cdot T \cdot T|)^2 = 0.125 \qquad P_2 \quad (|T \cdot R \cdot R \cdot T|)^2 = 0.0625 \qquad P_3 \quad (|T \cdot T \cdot M \cdot R|)^2 = 0.125$

$P_{12} \qquad P_{13} \qquad P_{23}$

$$(|R \cdot M \cdot T \cdot T + T \cdot R \cdot R \cdot T|)^2 + (|R \cdot M \cdot T \cdot T + T \cdot T \cdot M \cdot R|)^2 + (|T \cdot R \cdot R \cdot T + T \cdot T \cdot M \cdot R|)^2 = 1.2286$$

These calculations show that three-path interference is zero.

$$I_{123} := 0.9161 + 0.125 + 0.0625 + 0.125 - 1.2286 \rightarrow 0$$

This, of course, is a theoretical result based on the definitions of P_i and I_{ij} given above. The experimental results reported by Sinha *et al*. for the related triple-slit diffraction experiment are in agreement with the theoretical expectation to within experimental error

Objection to the Definitions

Sinha *et al*. write the double-slit wave function (unnormalized) as a linear superposition of the photon taking two paths (A and B) to the detector. The square modulus of the wave function gives the probability expression.

$$|\Psi|^2 = |\psi_A|^2 + |\psi_B|^2 + \psi_A^* \psi_B + \psi_B^* \psi_A$$

Obviously this is just traditional quantum mechanical mathematical procedure. The problem I have is with the interpretation or partition of this equation that comes next. The authors write the probability expression as

$$|\Psi|^2 = P_A + P_B + I_{AB}$$

where $P_A(P_B)$ is the probability that only slit A(B) is open and the remaining terms represent the actual interference.

I do not believe this sort of partitioning of the terms of $|\Psi|^2$ is quantum mechanically legitimate. If only slit A is open then the wave function is $\Psi = \psi_A$ and not $\Psi = \psi_A + \psi_B$. It is specious reasoning to say, in the double-slit experiment, that $|\psi_A|^2$ is the probability that the photon goes through slit A.

As Roy Glauber has written [AJP **63**, 12 (1995)], "The things that interfere in quantum mechanics ... are the probability amplitudes for certain events." The triple-slit experiment involves the interference of three probability amplitudes because there are three paths from the source (S) to each position (x) on the detection screen. According to quantum fundamentals, each photon takes all three paths simultaneously.

$$\Psi_{123} = \langle x|S\rangle = \langle x|3\rangle\langle 3|S\rangle + \langle x|2\rangle\langle 2|S\rangle + \langle x|1\rangle\langle 1|S\rangle$$