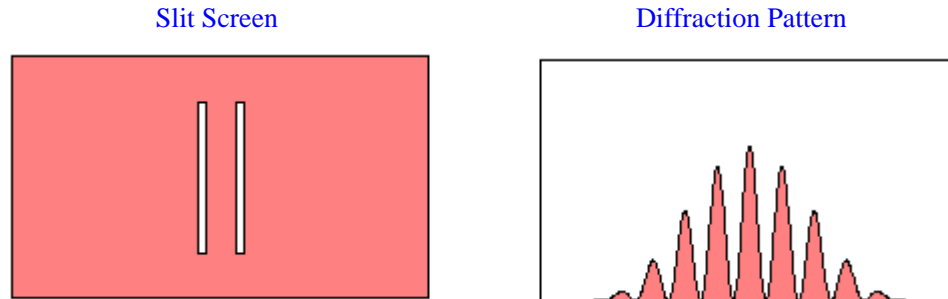


The Double-Slit Experiment

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The experiment: The slit screen on the left produces the diffraction pattern on the right when illuminated with a coherent radiation source.



The quantum mechanical explanation: Illumination of a double-slit screen with a coherent light source leads to a Schrodinger "cat state", in other words a superposition of the photon being localized at the both slits simultaneously.

$$|\Psi\rangle = \frac{1}{\sqrt{2}} [|x_1\rangle + |x_2\rangle]$$

Here x_1 and x_2 are positions of the two slits. It is assumed initially, for the sake of mathematical simplicity, that the slits are infinitesimally thin in the x-direction and infinitely long in the y-direction.

Because the slits localize the photon in the x-direction the uncertainty principle ($\Delta x \Delta p_x > \hbar/4\pi$) demands a compensating delocalization in the x-component of the momentum. To see this delocalization in momentum requires a momentum wave function, which is obtained by a Fourier transform of the position wave function given above.

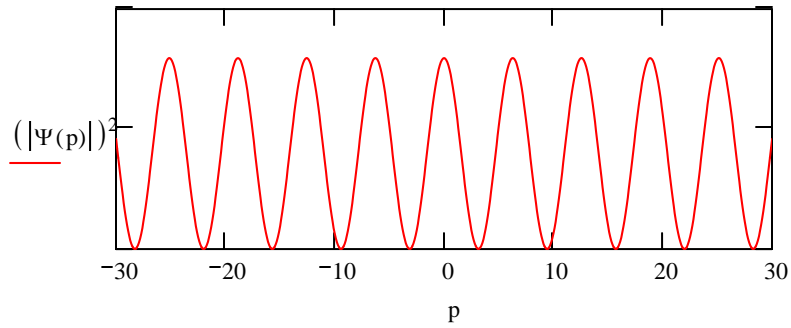
In this case the Fourier transform is simply the projection of the position wave function onto momentum space.

$$\langle p | \Psi \rangle = \frac{1}{\sqrt{2}} [\langle p | x_1 \rangle + \langle p | x_2 \rangle] = \frac{1}{\sqrt{2}} \left[\exp\left(-\frac{ipx_1}{\hbar}\right) + \exp\left(-\frac{ipx_2}{\hbar}\right) \right]$$

The quantum mechanical interpretation of the double-slit experiment, or any diffraction experiment for that matter, is that the diffraction pattern is actually the momentum distribution function, $|\langle p | \Phi \rangle|^2$. This is illustrated below.

Position of first slit: $x_1 := 0$ Position of second slit: $x_2 := 1$

$$p := -30, -29.9 .. 30 \quad \Psi(p) := \frac{\frac{1}{\sqrt{2 \cdot \pi}} \cdot \exp(-i \cdot p \cdot x_1) + \frac{1}{\sqrt{2 \cdot \pi}} \cdot \exp(-i \cdot p \cdot x_2)}{\sqrt{2}}$$



The momentum distribution shows interference fringes because the photons were localized in space at two positions: the maxima represent constructive interference and the minima destructive interference. Notice also that the momentum distribution in the x-direction is completely delocalized; it shows no sign of attenuating at large momentum values. This is due to the fact that under the present model the photons are precisely localized at x_1 and x_2 - in other words the slits are infinitesimally thin in the x-direction as specified above.

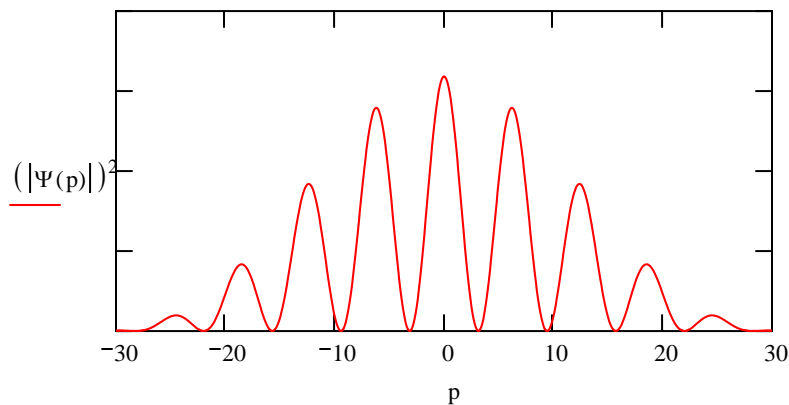
This of course is not an adequate representation of the actual double-slit diffraction pattern because any real slit has a finite size in both directions. It's really not a problem that the slit is infinite in the y-direction, because that means the photon is simply not localized in that direction. So all we need to do to get a more realistic double-slit diffraction pattern is make the slits finite (not infinitesimal) in the x-direction. This is accomplished below by giving the slits a finite width, δ , in the x-direction, and recalculating the momentum wave function.

Number of slits: $n := 2$ Slit positions: $j := 1..n$ $x_j := j$

$$\sum_{j=1}^n \int_{x_j - \frac{\delta}{2}}^{x_j + \frac{\delta}{2}} \frac{1}{\sqrt{2 \cdot \pi}} \cdot \exp(-i \cdot p \cdot x) \cdot \frac{1}{\sqrt{\delta}} dx$$

Slit width: $\delta := .2$

$$\Psi(p) := \frac{\quad}{\sqrt{n}}$$



In summary, the quantum mechanical interpretation of the double-slit experiment is that position is measured at the slit screen and momentum is measured at the detection screen. Position and momentum are conjugate observables that are connected by a Fourier transform and governed by the uncertainty principle. Knowing the slit screen geometry makes it possible to calculate the momentum distribution at the detection screen. Varying the slit width, δ , gives a clear and simple demonstration of the uncertainty principle in action. Narrow slit widths give broad momentum distributions and wide slit widths give narrow momentum distributions.

