The Double-Slit Experiment

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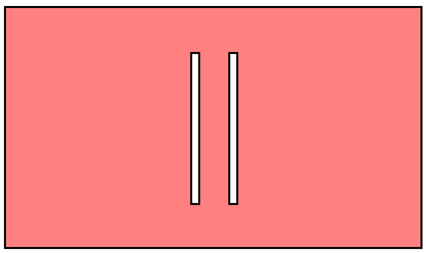
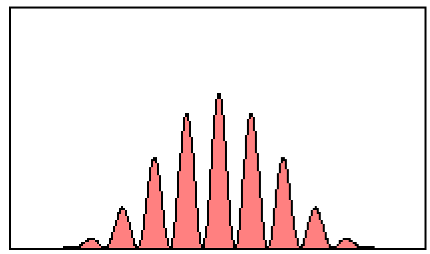
The experiment: The slit screen on the left produces the diffraction pattern on the right when illuminated with a coherent radiation source.

Slit Screen



Diffraction Pattern



The quantum mechanical explanation: Illumination of a double-slit screen with a coherent light source leads to a Schrodinger "cat state", in other words a superposition of the photon being localized at the both slits simultaneously.

$$\left|\Psi\right\rangle = \frac{1}{\sqrt{2}}\left[\left|x_1\right\rangle + \left|x_2\right\rangle\right]$$

Here x_1 and x_2 are positions of the two slits. It is assummed initially, for the sake of mathematical simplicity, that the slits are infinitesimally thin in the x-direction and infinitely long in the y-direction.

Because the slits localize the photon in the x-direction the uncertainty principle ($\Delta x \Delta p_x > h/4\pi$) demands a compensating delocalization in the x-component of the momentum. To see this delocalization in momentum requires a momentum wave function, which is obtained by a Fourier transform of the position wave function given above.

In this case the Fourier transform is simply the projection of the position wave function onto momentum space.

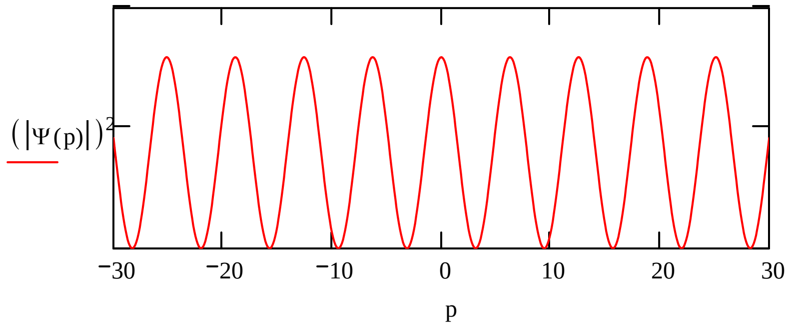
$$\left\langle p \middle| \Psi\right\rangle = \frac{1}{\sqrt{2}}\left[\left\langle p \middle| x_1\right\rangle + \left\langle p \middle| x_2\right\rangle\right] = \frac{1}{\sqrt{2}}\left[\exp\left(-\frac{ipx_1}{\hbar}\right) + \exp\left(-\frac{ipx_2}{\hbar}\right)\right]$$

The quantum mechanical interpretation of the double-slit experiment, or any diffraction experiment for that matter, is that the diffraction pattern is actually the momentum distribution function, $|\langle p \mid \Phi \rangle|^2$. This is illustrated below.

Position of first slit: $x_1 := 0$ Position of second slit: $x_2 := 1$

$p := -30, -29.9 .. 30$

$$\Psi(p) := \frac{\frac{1}{\sqrt{2\cdot\pi}}\cdot\exp(-i\cdot p\cdot x_1) + \frac{1}{\sqrt{2\cdot\pi}}\cdot\exp(-i\cdot p\cdot x_2)}{\sqrt{2}}$$



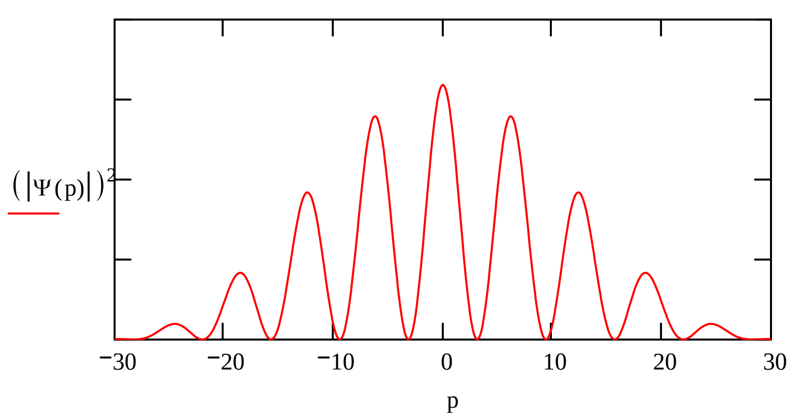
The momentum distribution shows interference fringes because the photons were localized in space at two positions: the maxima represent constructive interference and the minima destructive interference. Notice also that the momentum distribution in the x-direction is completely delocalized; it shows no sign of attenuating at large momentum values. This is due to the fact that under the present model the photons are precisely localized at x_1 and x_2 - in other words the slits are infinitesimally thin in the x-direction as specificed above.

This of course is not an adequate representation of the actual double-slit diffraction pattern because any real slit has a finite size in both directions. It's really not a problem that the slit is infinite in the y-direction, because that means the photon is simply not localized in that direction. So all we need to do to get a more realistic double-slit diffraction pattern is make the slits finite (not infinitesimal) in the x-direction. This is accomplished below by giving the slits a finite width, δ, in the x-direction, and recalculating the momentum wave function.

Number of slits: $n := 2$ Slit positions: $j := 1 .. n$ $x_j := j$

Slit width: $\delta := .2$

$$\Psi(p) := \frac{\displaystyle\sum_{j=1}^{n} \int_{x_j-\frac{\delta}{2}}^{x_j+\frac{\delta}{2}} \frac{1}{\sqrt{2\cdot\pi}}\cdot\exp(-i\cdot p\cdot x)\cdot\frac{1}{\sqrt{\delta}}\,dx}{\sqrt{n}}$$



In summary, the quantum mechanical interpretation of the double-slit experiment is that position is measured at the slit screen and momentum is measured at the detection screen. Position and momentum are conjugate observables that are connected by a Fourier transform and governed by the uncertainty principle. Knowing the slit screen geometry makes it possible to calculate the momentum distribution at the detection screen. Varying the slit width, δ, gives a clear and simple demonstration of the uncertainty principle in action. Narrow slit widths give broad momentum distributions and wide slit widths give narrow momentum distributions.