The Temporal Double-Slit Experiment

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In the familiar spatial double-slit experiment illumination of the slit screen leads to a "Schrodinger cat state" in which photons or other quons are in a linear superposition of being present at both slits simultaneously.

\[ |\Psi\rangle = \frac{1}{\sqrt{2}} \left[ |x_1\rangle + |x_2\rangle \right] \]

This spatial superposition leads to interference fringes at the detection screen. See for example the previous tutorial (http://www.users.csbsju.edu/~frioux/two-slit/another2slit.pdf).

Recently Physics World reported a temporal version of the double-slit experiment performed by an international team led by Gerhard Paulus (http://www.physicsweb.org/articles/news/9/3/1/1). In this remarkable experiment a 5-femtosecond laser pulse, which consists of two maxima and one minimum in the electric field, is used to ionize argon atoms.

![Image of a laser pulse and ionized photons]

As shown in the figure below, the maxima and minimum accelerate the ionized photons in opposite directions.

According to quantum mechanics the electrons traveling to the right-hand detector are represented by a linear superposition of being ionized at two different times, say \( t_1 \) and \( t_2 \).

\[ |\Psi\rangle = \frac{1}{\sqrt{2}} \left[ |t_1\rangle + |t_2\rangle \right] \]
At the detector Paulus et al. measure the kinetic energy of the electrons and observe the interference fringes shown in the figure above. The fringes can be understood by projecting this linear superposition onto the energy representation.

\[
\langle E \mid \Psi \rangle = \frac{1}{\sqrt{2}} \left[ \langle E \mid t_1 \rangle + \langle E \mid t_2 \rangle \right] = \frac{1}{\sqrt{2}} \left[ \exp\left( -\frac{iEt_1}{\hbar} \right) + \exp\left( -\frac{iEt_2}{\hbar} \right) \right]
\]

Clearly, there are two temporal "paths" for each observed energy value and they interfere with each other constructively and destructively. Plotting \( |\langle E \mid \Psi \rangle|^2 \) shows that this is the origin of the observed fringes.

Times of maxima: \( t_1 := 1 \quad t_2 := 5 \quad \Psi(E) := \frac{1}{\sqrt{2}} \cdot \sqrt{\pi} \cdot \exp(-i \cdot E \cdot t_1) + \frac{1}{\sqrt{2}} \cdot \sqrt{\pi} \cdot \exp(-i \cdot E \cdot t_2) \)

This calculation shows no attenuation in the fringes at the extremes of energy because there is no uncertainty in the time of the pulse maxima. A calculation which introduces an uncertainty of \( \delta \) in the duration of each maximum yields a more realistic picture of the interference fringes.

\[
\delta := .5 \quad \Psi(E) := \frac{1}{\sqrt{2}} \cdot \sqrt{\pi} \cdot \left[ \int_{t_1}^{t_1 + \frac{\delta}{2}} \exp(-i \cdot E \cdot t) \, dt + \int_{t_2}^{t_2 + \frac{\delta}{2}} \exp(-i \cdot E \cdot t) \, dt \right]
\]

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Relevant references:
