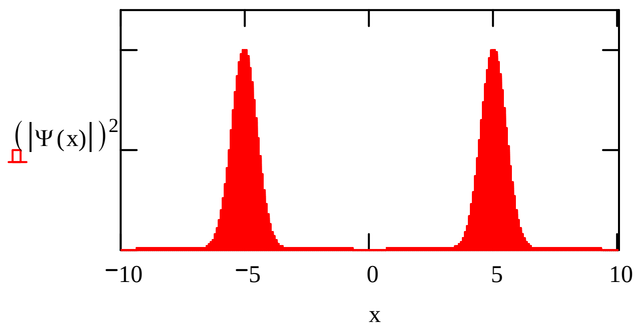
Wigner Distribution for the Double Slit Experiment

Illumination of a double slit with a coherent incident particle beam leads to a "cat state" which can be represented by a linear superposition of two Gaussian wavepackets as shown below.

$x := -10, -9.99 .. 10 \qquad \Psi(x) := \exp\left[-(x-5)^2\right] + \exp\left[-(x+5)^2\right]$

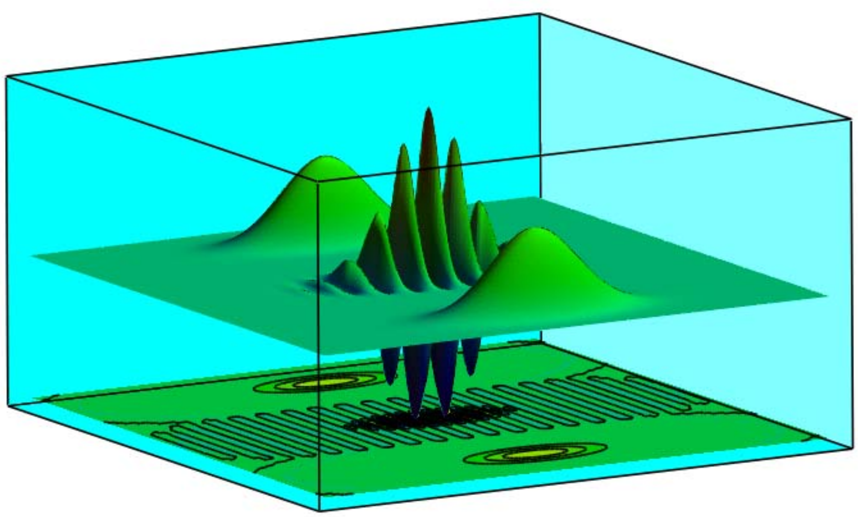


The Wigner distribution for this function is calculated and plotted below.

$$W(x,p) := \frac{1}{2\cdot\pi}\cdot\int_{-\infty}^{\infty}\left[\exp\left[-\left(x+\frac{s}{2}-5\right)^2\right]+\exp\left[-\left(x+\frac{s}{2}+5\right)^2\right]\right]\cdot\exp(i\cdot p\cdot s)\cdot\left[\exp\left[-\left(x-\frac{s}{2}-5\right)^2\right]+\exp\left[-\left(x-\frac{s}{2}+5\right)^2\right]\right]ds$$

$$W(x,p) := \frac{1}{\sqrt{2\cdot\pi}}\cdot\left(2\cdot\exp\left(-2\cdot x^2-\frac{1}{2}\cdot p^2\right)\cdot\cos(10\cdot p)+\exp\left(-2\cdot x^2+20\cdot x-50-\frac{1}{2}\cdot p^2\right)+\exp\left(-2\cdot x^2-20\cdot x-50-\frac{1}{2}\cdot p^2\right)\right)$$

$N := 100 \qquad i := 0..N \qquad x_i := -7+\frac{14\cdot i}{N} \qquad j := 0..N \qquad p_j := -6+\frac{12\cdot j}{N} \qquad Wigner_{i,j} := W(x_i, p_j)$



Wigner , Wigner

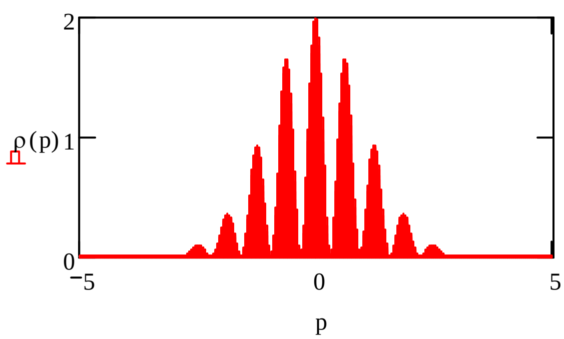
The Wigner distribution can be reconstructed from experimental measurements using quantum state tomography. Reconstructive tomography is a widely used technique in medicine, for example, for obtaining the shape of an inaccessible two-dimensioanl object from a set of different one-dimensional "shadows" cast by that object.

Quantum state reconstruction is possible if a system can be prepared repeatedly in the same state. Subsequent measurements on such a system are then effectively mutltiple measurements on the same quantum state. The theoretical Wigner distribution shown above for the double-slit experiment has been recently reconstructed for the helium atom. [See, *Nature*, **386**, 150 (1997).

Integration of the Wigner distribution for the linear superposition over the spatial coordinate yields the momentum distribution function.

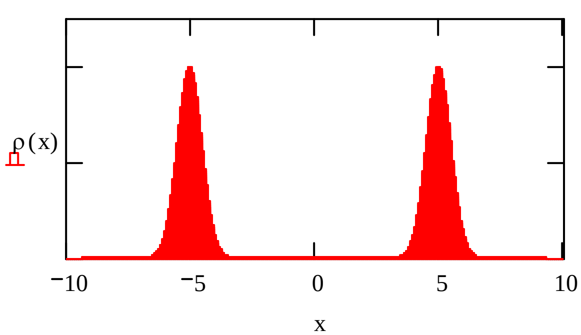
$$\rho(p) := \int_{-\infty}^{\infty} \frac{1}{\sqrt{2\cdot\pi}}\cdot\left(2\cdot\exp\left(-2\cdot x^2 - \frac{1}{2}\cdot p^2\right)\cdot\cos(10\cdot p) + \exp\left(-2\cdot x^2 + 20\cdot x - 50 - \frac{1}{2}\cdot p^2\right) \ldots \atop + \exp\left(-2\cdot x^2 - 20\cdot x - 50 - \frac{1}{2}\cdot p^2\right)\right) dx$$

$$\rho(p) := 2\cdot\left(\frac{1}{2}\cdot\cos(10\cdot p) + \frac{1}{2}\right)\cdot\exp\left(\frac{-1}{2}\cdot p^2\right) \qquad p := -5, -4.99 .. 5$$



Integration of the Wigner distribution for the linear superposition over the momentum coordinate returns the spatial distribution function.

$$\rho(x) := \int_{-4}^{4} \frac{1}{\sqrt{2\cdot\pi}}\cdot\left(2\cdot\exp\left(-2\cdot x^2 - \frac{1}{2}\cdot p^2\right)\cdot\cos(10\cdot p) + \exp\left(-2\cdot x^2 + 20\cdot x - 50 - \frac{1}{2}\cdot p^2\right) \ldots \atop + \exp\left(-2\cdot x^2 - 20\cdot x - 50 - \frac{1}{2}\cdot p^2\right)\right) dp \qquad x := -10, -9.99 .. 10$$



See Figure 1 of "Shadows and Mirrors:Reconstructing Quantum States of Atom Motion," Physics Today, April 1998, by Leibfried, Pfau, and Monroe.

Reference: Decoherence and the Transition form Quantum to Classical, Wojciech Jurek, Physics Today, October 1991, pages 36-44.