The Wigner Function for the Single Slit Diffraction Problem
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The quantum mechanical interpretation of the single-slit experiment is that position is measured at the slit screen and momentum is measured at the detection screen. Position and momentum are conjugate observables connected by a Fourier transform and governed by the uncertainty principle. Knowing the slit screen geometry makes it possible to calculate the momentum distribution at the detection screen.

The slit-screen geometry and therefore the coordinate wavefunction is calculate as follows.

Slit width: \( w := 2 \)  
Coordinate-space wave function: \( \Psi(x, w) := \text{if} \begin{cases} x \geq -\frac{w}{2}, & x \leq \frac{w}{2}, \\ 1, & 0 \end{cases} \)

\[
x := -\frac{w}{2}, \frac{w}{2} + .005 .. \frac{w}{2}
\]

A Fourier transform of the coordinate-space wave function yields the momentum wave function and the momentum distribution function, which is the diffraction pattern.

\[
\Phi(p_x, w) := \frac{1}{\sqrt{2 \cdot \pi \cdot w}} \int_{-\frac{w}{2}}^{\frac{w}{2}} \exp(-i \cdot p_x \cdot x) \, dx \text{ simplify to} 2^{\frac{1}{2}} \cdot \frac{\sin \left( \frac{1}{2} \cdot w \cdot p_x \right)}{2 \cdot \pi \cdot w \cdot p_x}
\]

\[
\left( \frac{1}{\Delta} \Phi(p_x, w) \right)^2
\]

The Wigner function for the single-slit screen geometry is generated using the momentum wave function. (Fifty is effectively infinity and is therefore as the limits of integration.)

\[
W(x, p) := \frac{1}{2 \cdot \pi} \int_{-50}^{50} \Phi \left( p + \frac{s}{2} \cdot w \right) \cdot \exp(-i \cdot s \cdot x) \cdot \Phi \left( p - \frac{s}{2} \cdot w \right) \, ds
\]
The single-slit Wigner function is displayed graphically.

\[
\begin{align*}
N &:= 150 & i &:= 0..N & x_i &:= -1.5 + \frac{3 \cdot i}{N} & j &:= 0..N & p_j &:= -20 + \frac{40 \cdot j}{N} & Wigner_{i,j} &:= W(x_i, p_j)
\end{align*}
\]