Superposition vs. Mixture

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The Wigner function can be used to illustrate the difference between a superposition and a mixture. First consider the following linear superposition of Gaussian functions.

$$\Psi(x) := \exp\left[-(x-5)^2\right] + \exp\left[-(x+5)^2\right]$$

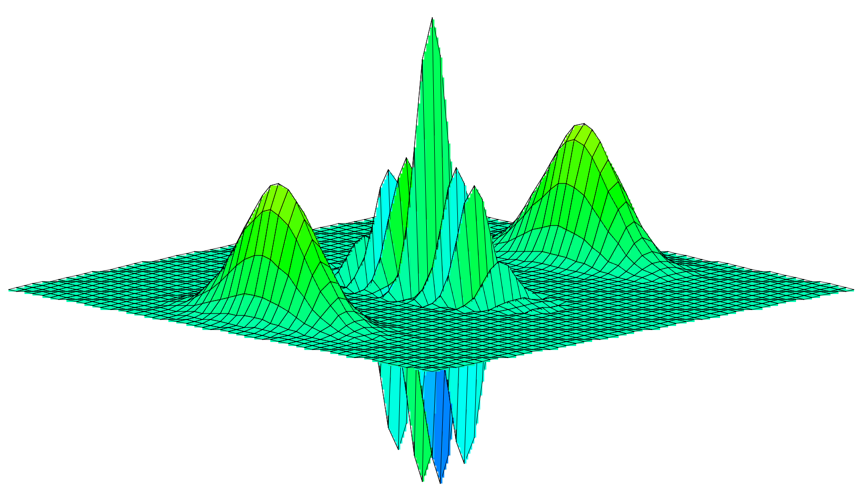
The Wigner distribution for this function is calculated and plotted below.

$$W(x,p) := \int_{-\infty}^{\infty} \left[\exp\left[-\left(x+\frac{s}{2}-5\right)^2\right] + \exp\left[-\left(x+\frac{s}{2}+5\right)^2\right]\right]\cdot \exp(i\cdot p\cdot s)\cdot \left[\exp\left[-\left(x-\frac{s}{2}-5\right)^2\right] + \exp\left[-\left(x-\frac{s}{2}+5\right)^2\right]\right] ds$$

Integration yields:

$$W(x,p) := \sqrt{2}\cdot\sqrt{\pi}\cdot\left(2\cdot\exp\left(-2\cdot x^2 - \frac{1}{2}\cdot p^2\right)\cdot\cos(10\cdot p) + \exp\left(-2\cdot x^2 + 20\cdot x - 50 - \frac{1}{2}\cdot p^2\right) + \exp\left(-2\cdot x^2 - 20\cdot x - 50 - \frac{1}{2}\cdot p^2\right)\right)$$

$N := 50 \qquad i := 0 .. N \qquad x_i := -7 + \frac{14\cdot i}{N} \qquad j := 0 .. N \qquad p_j := -6 + \frac{12\cdot j}{N} \qquad \text{Wigner}_{i,j} := W(x_i, p_j)$



Wigner

The signature of a superposition is the occurence of interference fringes as seen in the center of the figure above.

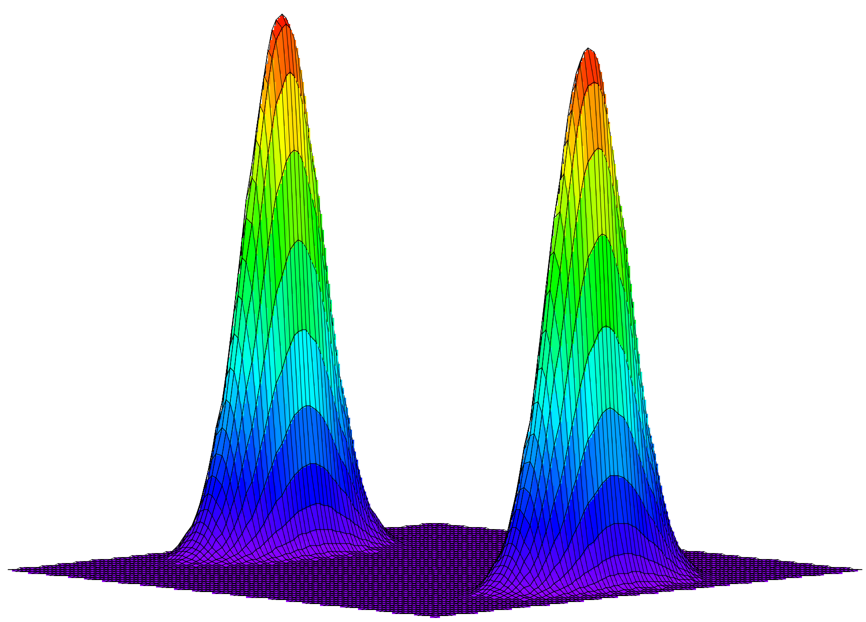
The Wigner function for a classical mixture is the sum of Wigner functions for each member of the mixture. The interference region is clearly absent in the figure shown below.

$$W(x,p) := \int_{-\infty}^{\infty} \exp\left[-\left(x+\frac{s}{2}-5\right)^2\right]\cdot\exp(i\cdot p\cdot s)\cdot\exp\left[-\left(x-\frac{s}{2}-5\right)^2\right] ds \ldots$$
$$+ \int_{-\infty}^{\infty} \exp\left[-\left(x+\frac{s}{2}+5\right)^2\right]\cdot\exp(i\cdot p\cdot s)\cdot\exp\left[-\left(x-\frac{s}{2}+5\right)^2\right] ds$$

Integration yields:

$$W(x,p) := \exp\left(-2\cdot x^2+20\cdot x-50-\frac{1}{2}\cdot p^2\right)\cdot\sqrt{2}\cdot\sqrt{\pi}+\exp\left(-2\cdot x^2-20\cdot x-50-\frac{1}{2}\cdot p^2\right)\cdot\sqrt{2}\cdot\sqrt{\pi}$$

$N := 100$ $\quad i := 0..N$ $\quad x_i := -7+\frac{14\cdot i}{N}$ $\quad j := 0..N$ $\quad p_j := -6+\frac{12\cdot j}{N}$ $\quad Wigner_{i,j} := W(x_i, p_j)$



Wigner

Reference: Decoherence and the Transition form Quantum to Classical, Wojciech Jurek, Physics Today, October 1991, pages 36-44.