The de Broglie-Bohr Model for the Hydrogen Atom

\[ \lambda = \frac{h}{m \cdot v} \]
de Broglie’s hypothesis that matter has wave-like properties.

\[ n \cdot \lambda = 2 \cdot \pi \cdot r \]
The consequence of de Broglie’s hypothesis; an integral number of wavelengths must fit within the circumference of the orbit. This introduces the quantum number which can have values 1, 2, 3, ...

\[ m \cdot v = \frac{n \cdot h}{2 \cdot \pi \cdot r} \]
Substitution of the first equation into the second equation reveals that linear momentum is quantized.

\[ T = \frac{1}{2} m \cdot v^2 = \frac{n^2 \cdot h^2}{8 \cdot \pi^2 \cdot m_e \cdot r^2} \]
If momentum is quantized, so is kinetic energy.

\[ E = T + V = \frac{n^2 \cdot h^2}{8 \cdot \pi^2 \cdot m_e \cdot r^2} - \frac{e^2}{4 \cdot \pi \cdot \epsilon_0 \cdot r} \]
Which means that total energy is quantized.

Below the ground state energy and orbit radius of the electron in the hydrogen atom is found by plotting the energy as a function of the orbital radius. The ground state is the minimum in the curve.

Fundamental constants: electron charge, electron mass, Planck’s constant, vacuum permitivity.

\[ e := 1.602177 \cdot 10^{-19} \text{ coul} \]
\[ m_e := 9.10939 \cdot 10^{-31} \text{ kg} \]
\[ h := 6.62608 \cdot 10^{-34} \text{ joule \cdot sec} \]
\[ \epsilon_0 := 8.85419 \cdot 10^{-12} \frac{\text{coul}^2}{\text{joule \cdot m}} \]

Quantum number and conversion factor between meters and picometers and joules and atto joules.

\[ n := 1 \]
\[ \text{pm} := 10^{-12} \text{ m} \]
\[ \text{atto joule} := 10^{-18} \text{ joule} \]

\[ r := 20 \text{ pm}, 20.5 \text{ pm} \ldots 500 \text{ pm} \]

\[ T(r) := \frac{n^2 \cdot h^2}{8 \cdot \pi^2 \cdot m_e \cdot r^2} \]
\[ V(r) := \frac{e^2}{4 \cdot \pi \cdot \epsilon_0 \cdot r} \]
\[ E(r) := T(r) + V(r) \]

This figure shows that atomic stability involves a balance between potential and kinetic energy. The electron is drawn toward the nucleus by the attractive potential energy interaction (\(\sim -1/R\)), but is prevented from spiraling into the nucleus by the extremely large kinetic energy (\(\sim 1/R^2\)) associated with small orbits.

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