The de Broglie-Bohr Model for the Hydrogen Atom

\[ \lambda = \frac{h}{m \cdot v} \]

de Broglie's hypothesis that matter has wave-like properties.

\[ n \cdot \lambda = 2 \cdot \pi \cdot r \]

The consequence of de Broglie's hypothesis; an integral number of wavelengths must fit within the circumference of the orbit. This introduces the quantum number, \( n \), which can have values 1, 2, 3, ...

\[ m \cdot v = \frac{n \cdot h}{2 \cdot \pi \cdot r} \]

Substitution of the first equation into the second equation reveals that linear momentum is quantized.

\[ T = \frac{1}{2} m \cdot v^2 = \frac{n^2 \cdot h^2}{8 \cdot \pi^2 \cdot m_e \cdot r^2} \]

If momentum is quantized, so is kinetic energy.

\[ E = T + V = \frac{n^2 \cdot h^2}{8 \cdot \pi^2 \cdot m_e \cdot r^2} - \frac{q^2}{4 \cdot \pi \cdot \varepsilon_0 \cdot r} \]

Which means that total energy is quantized.

Below the ground state energy and orbit radius of the electron in the hydrogen atom is found by plotting the energy as a function of the orbital radius. The ground state is the minimum in the curve.

Fundamental constants: electron charge, electron mass, Planck's constant, vacuum permittivity.

\[ q := 1.6021777 \cdot 10^{-19} \text{ coul} \quad m_e := 9.10939 \cdot 10^{-31} \text{ kg} \]

\[ h := 6.62608 \cdot 10^{-34} \text{ joule sec} \quad \varepsilon_0 := 8.85419 \cdot 10^{-12} \frac{\text{ coul}^2}{\text{ joule m}} \]

Conversion factors between meters and picometers and joules and atto joules.

\[ pm := 10^{-12} \text{ m} \quad \text{ajoule} := 10^{-18} \text{ joule} \quad eV := 1.602177 \cdot 10^{-19} \text{ joule} \]

Setting the first derivative of the energy with respect to \( r \) equal to zero, yields the optimum value of \( r \).

\[ \frac{d}{dr} \left( \frac{n^2 \cdot h^2}{8 \cdot \pi^2 \cdot m_e \cdot r^2} - \frac{q^2}{4 \cdot \pi \cdot \varepsilon_0 \cdot r} \right) = 0 \]

has solution(s)

\[ \frac{n^2 \cdot h^2}{q^2} \cdot \frac{\varepsilon_0}{\pi \cdot m_e} \]

Substitution of this value of \( r \) back into the energy expression yields the energy gives the energy of the hydrogen atom in terms of the quantum number, \( n \), and the fundamental constants.

\[ E = \frac{n^2 \cdot h^2}{8 \cdot \pi^2 \cdot m_e \cdot r^2} - \frac{q^2}{4 \cdot \pi \cdot \varepsilon_0 \cdot r} \]

by substitution, yields

\[ E = \frac{-1}{8 \cdot n^2 \cdot h^2} \cdot \frac{m_e}{\varepsilon_0^2} \cdot q^4 \]

Calculate the allowed energy levels for the hydrogen atom:

\[ n := 1, 2, 3, 4 \]

\[ E_n := \frac{-1}{8 \cdot n^2 \cdot h^2} \cdot \frac{m_e}{\varepsilon_0^2} \cdot q^4 \]

\[ \begin{align*}
E_n \text{ [ajoule]} &= \begin{pmatrix} -2.18 \\ -0.545 \\ -0.242 \\ -0.136 \\ -0.087 \end{pmatrix} \\
E_n \text{ [eV]} &= \begin{pmatrix} -13.606 \\ -3.401 \\ -1.512 \\ -0.85 \\ -0.544 \end{pmatrix}
\end{align*} \]

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